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SOME FUNDAMENTAL CONCEPTS OF STATISTICS[1]

By Warren M. Persons

The general topic for discussion at the present annual meeting of the American Statistical Association is, "the statistical basis for analyzing the current economic situation, with the object of making forecasts of business conditions in general and for the great groups of economic activities in the United States." This topic explicitly recognizes that statistics may be effectively utilized not merely to describe the past, but as a basis for estimating present and future tendencies. The point of view thus indicated is neither new nor revolutionary. When the Statistical Society of London was organized in 1834, five years before our own Association, the Prospectus announced that its functions were to "procure, arrange, and publish facts calculated to illustrate the conditions *and prospects* of society." The evaluation of "the prospects of society" was thus recognized in this early statement as a proper object of statistical research.

It was not by accident that the idea of utilizing statistics of the past condition of society for inferences concerning the future appeared in the Prospectus of 1834. That it was expressed deliberately and with comprehension of its significance is indicated by an interesting and pertinent incident in the organization of the London Society, related by its former president, Sir Athelstone Baines. A Statistical Section of the British Association for the Advancement of Science had been organized in 1833 in which the studies were limited to "Facts relating to communities of men which are capable of being expressed by numbers, and which promise, when sufficiently multiplied, to indicate general laws." "Several men of eminence on statistics," says Sir Athelstone, "chafed at being thus relegated to the position of 'hewers and

[1] The Presidential address read at the Eighty-fifth Annual Meeting of the American Statistical Association, Washington, D. C., December, 1923.

drawers for political economy and philosophy,' so they joined in promoting the Statistical Society of London, now the Royal Statistical Society, with the view of providing therein a wider scope for their inquiries." [1] They considered it to be the function of *the statistician* to interpret and draw inferences from his statistics, and organized the Statistical Society for the purpose of furthering the development of that function.

It is my object tonight to discuss some fundamental concepts of statistics. I will consider, particularly, the logical significance of a statistical exhibit, the nature of statistical inference, and some of the important concepts involved in making forecasts of economic conditions on the basis of statistical analyses of limited periods in the past.

The necessity of the accumulation of statistics of the complex world of affairs in which we are immersed and the equal necessity of the development of special methods, different from those of the exact sciences, for summarizing these data have been admirably expressed by John T. Merz in his *History of European Thought in the Nineteenth Century*. "That which everywhere oppresses the practical man," he says, "is the great number of things and events which pass ceaselessly before him, and the flow of which he cannot arrest. What he requires is the grasp of large numbers. The successful scientific explorer has always been the man who could single out some special thing for minute and detailed investigation, who could retire with one definite object, with one fixed problem into his study or laboratory and there fathom and unravel its intricacies, rising by induction or divination to some rapid generalization which allowed him to establish what is termed a law or general aspect from which he could view the whole or a large part of nature. The scientific genius can 'stay the moment fleeting. . . .' The practical man cannot do this; he is always and everywhere met by the crowd of facts; by the relentlessly hurrying stream of events. What he requires is grasp of numbers, leaving to the professional man the knowledge of detail. Thus has arisen the science of large numbers or statistics, and the many methods of which it is possessed." [2]

The contrast, indicated by Merz, between the experimental method of the natural sciences and the statistical method of the social sciences may be expressed in greater detail. The natural scientist sets up and tries his experiment; repeats it as often as he pleases under the same or varying conditions; isolates the factor in which he is interested; and arrives at a demonstrable conclusion concerning the operation and effects of that factor. The social scientist, on the other hand, must ac-

[1] *The History of Statistics*, edited by John Koren, p. 385.
[2] Vol. II, pp. 554–5.

cept and analyze the mixed situation as it comes to him; gather pertinent statistics, not such as he would like, but such as are available; study figures which embody the combined effects of many factors; and express his conclusions in terms of probabilities.

It is obvious that the scope and nature of statistical inquiries are determined, in the first instance, by the extent and kind of records yielded by the "relentlessly hurrying stream of events". Many of these records, which constitute the material for statistical research, have resulted by accident rather than by design. For instance, in the seventeenth century the French government sold to a group of purchasers, bonds containing the provision that upon the death of any holder the income from his bond was to be distributed to the surviving holders; the record of this bond issue constituted the material for one of the first tables of mortality. But we need not go back to the seventeenth century for illustrations of the accidental origin of useful statistics. Our figures for the personal distribution of incomes are a by-product of tax administration; bank clearings are a consequence of the process of check collection; and figures for building permits have resulted from the regulation of urban construction. It is only in comparatively recent times that such statistical by-products have been supplemented by special collections of data definitely planned, and designed to answer specific social or economic questions. And it is a still more recent development for various organizations, such as journals, trade associations, and governmental agencies, to collect and publish current data expressly for the purpose of estimating tendencies with the object of moulding private and public policies. The body of new statistics which has been added to our store during the five years since the war shows that we are making rapid progress toward securing a more adequate statistical record of "the relentlessly hurrying stream of events."

The boundaries of statistical research set by the material available, are obviously being widened, particularly in the field of economics. But it is not now my purpose to consider either these widening boundaries or their limitations upon research. My purpose, as I have said, is to consider the nature of the argument which proceeds from an examination of series of economic statistics, such as prices, interest rates, and the output of goods for consecutive time-units in a given period.

The primary function of an investigation of the economic statistics of a selected period of time is purely descriptive of that period.[1] Statistical devices, such as tables, charts, averages, measures of dispersion, lines or curves of trend, periodic functions, and coefficients of correla-

[1] See Keynes, *A Treatise on Probability*, p. 327.

tion, are not arguments. All of this machinery enables one merely to describe more or less completely and, at the same time, more or less simply, the economic fluctuations and inter-relations of the given period. The inclusion of such a function as the coefficient of correlation among the descriptive devices of statistics indicates and emphasizes our point of view. Keynes accurately characterizes the last named statistical device in these words, "the mere existence of a particular correlation coefficient as descriptive of a group of observations, even of a large group, is not in itself a more conclusive or significant argument than the mere existence of a particular frequency coefficient would be." [1] And a frequency coefficient, such as average wholesale prices or the ratio between bank loans and deposits, is of itself no argument at all. So that the immediate result of any research in social statistics is merely the description of a limited field.

But the statistician whom we are considering has not as his object the securing of an isolated description in a world of isolated descriptions. His aim is to draw an inference. In the specific case which we are considering at the present annual meeting of the American Statistical Association his object is the definite one of getting a notion of the economic "prospects of society". To attain his end he has selected a suitable body of statistics and a period for study. Back of this selection, back of the study itself, is a fundamental belief. The statistician, in common with every other scientific investigator, has a deep-seated belief in the continuity and orderliness of affairs.

"The grasp of large numbers," says Merz, "the methodical array of figures and the registration of events, would in itself be of little use were it not for a fundamental assumption which appeals to common sense and has been confirmed by science, though it is hardly anywhere expressly stated—namely, the belief in a general order, in a recurrent regularity or a slow but continuous change and orderly development of the things and events of the world. . . . It may also be well to note that this belief in a general order is common to all schools of thought." [2]

The recognition of this belief in a general orderliness of affairs is necessary if we are to understand the manner in which the statistician moulds his investigation and arrives at a statistical inference. In the first place, he follows, as well as his material allows, the method of the experimental scientist when he selects, as a basis for forecasting, a past period for study as nearly as possible like that of the present. He attempts to find a specific analogy existing in an orderly universe. But

[1] *A Treatise on Probability*, p. 426.
[2] *History of European Thought in the Nineteenth Century*, Vol. II, p. 556.

he realizes that analogies differ greatly in their persuasive quality. The importance for an inference of a given statistical result pertaining to a given period is greatly increased, first, if similar or consistent statistical results obtain for sub-periods; second, if similar or consistent statistical results obtain for other periods and under different circumstances; and third, if all of the statistical results agree with, are supported by, or can be set in the framework of, related knowledge of a statistical or non-statistical nature. To illustrate the first point, if we have found, for instance, that a periodic function with a period of 40 months fits a time series of money rates for a span of 50 years the conclusion that there is a real period of 40 months for the entire span is strengthened if we obtain the same periodic function for each of two or more segments of the given 50 years. Also, to illustrate the second point, the conclusion is further strengthened if the same function is found for other than the given 50-year span and its segments. Likewise, the conclusion of a 40-month period would be further supported by the securing of evidence, statistical or otherwise, of corresponding fluctuations in business affairs. In other words, stability of statistical results and agreement with non-statistical results are potent arguments for continued stability in an orderly universe.

The method of argument just described is essentially empirical. Oftentimes when arguments from statistics are so characterized the word "empirical" is used to classify statistical inference as a lower, and hardly respectable, order of human thought. It is not in this way that I use the term. For I conceive that the term empirical "means simply what belongs to or is the product of experience or observation," and that a stable empirical statistical result persisting over the entire range of our experience is precisely the same thing as a "law of nature."

You will say to me, "There is nothing novel in the type of argument which you ascribe to the statistician; it is induction, neither more nor less." To this, I assent. "But the statistician," you may continue, "unlike other investigators deals with large numbers relating to classes of phenomena, in which, though it is impossible to predict what will happen in an individual case, there is nevertheless a significant regularity of occurrence if the phenomena be considered in successive sets. From this regularity," you say, "and from the likeness of the stream of human events to games of chance, the mathematician has built up a technique of statistical probability. In this technique a given statistical series becomes a random sample and the various statistical devices used, such as averages and coefficients of correlation, consequently have determinable probable errors. From these probable errors we

can state precisely the future result which we may reasonably expect. For instance," you continue, "the maximum coefficient of correlation between the monthly items of pig-iron production and money rates for the period January 1903–July 1914 (133 pairs of items), after having made due allowance for secular trend and seasonal influences, is for a lag of 5 or 6 months in money rates and amounts to +0.75. The probable error of this coefficient is 0.03. The coefficient of correlation and the probable error together mean, according to the theory of inverse probability, that if we should take another sample of equal size the chances are equal that the resulting coefficient would be between +0.72 and +0.78. That is to say, one can state the mathematical probability that given economic relations will obtain in the future. This," you conclude, "is the special contribution of mathematical statistics to forecasting."

The view that the mathematical theory of probability provides a method of statistical induction or aids in the specific problem of forecasting economic conditions, I believe, is wholly untenable. Let us take a simple case. Suppose we are considering the probability that 1924 will be a year of business depression and that we have the record of business conditions for the past 100 years in which there were 40 years of depression and 60 years of non-depression. Then, according to the statistical record, the probability that a year taken at random would be depressed is 4/10. But this probability cannot refer to 1924 unless that year is a random year, that is, unless we have no specific information that differentiates it from other years. In fact, we do have specific information about the economic conditions in 1923 and about the relation of economic conditions in consecutive years, which we cannot reasonably ignore, so that we cannot view 1924 as "any" year taken at random. Every item of knowledge which we obtain bearing upon the situation in 1923 removes not only that year, but also 1924 one step further from its classification as "random." Such items of knowledge do not lead merely to continued revisions of the numerical probability that next year will be a year of depression; rather, they render inapplicable the method of mathematical probability, to the problem of making a rational forecast. Moreover, the actual statistical data utilized as a basis for forecasting economic conditions, such as a given time series of statistics for a selected period in the past, cannot be considered a random sample except in an unreal, hypothetical sense; that is to say, unless assumptions be made concerning our material which cannot be retained in actual practice. Any past period that we select for study is, in fact, a special period with characteristics distinguishing it from other periods, and is not

"random" with respect to the present. We must, therefore, discard statistical probability and arrive at a forecast for 1924 on another basis.

There is a special objection to the application of the theory of probability to the particular economic data which constitute our material. If the theory of probability is to apply to our data, not merely the series but the individual items of the series must be a random selection. In fact, a group of successive items with a characteristic conformation constitutes our material. Since the individual items are not independent, the probable errors of the constants of a time series, computed according to the usual formulas, do not have their usual mathematical meaning. Thus, the "probable error" of 0.03 in a coefficient of correlation of +0.75 between the monthly items of pig-iron production and money rates six months later does not indicate, as one would conclude from the theory of probability, that the chances are billions to one against the independence of the two variables; or, to state the idea more specifically, that if we compute a coefficient from data of "any" other actual period the chances are more than ten millions to one that its value would be over +0.50. In fact, the significance of the "probable error" of a constant computed from time series is not known, and, in practice, we do not view the world from the standpoint of mathematical probability. So that we are not surprised when we actually find that the coefficient of correlation between the adjusted figures for pig-iron production and money rates six months later for the period 1915–18 is only +0.38. We find sufficient explanation of this result, which is almost impossible and really astounding when viewed from the standpoint of random sampling, in the war demands for pig-iron, the tremendous imports of gold, government financing, and the inauguration of the federal reserve system during the period in question. Neither are we surprised when we find that for the period 1919–23 the maximum correlation between the two series is for a lag in money rates, not of six months, but of nine to twelve months. For this period includes the severe crisis and great financial stringency of 1920–21, which dominated most of the items and hence the results. Thus in actual practice the statistician cannot reasonably assume ignorance of the peculiar circumstances pertaining to the special cases which constitute his material, and therefore he does not think in terms of random sampling and numerical probabilities. Granting as one must that consecutive items of a statistical time series are, in fact, related makes inapplicable the mathematical theory of probability.

The thesis that statistical probabilities give us no aid in arriving at a statistical inference has been developed with great skill and, I

think, success by John Maynard Keynes in his *Treatise on Probability*, which I have already quoted. Summarizing his position, he says, "In order to get a good scientific argument we still have to pursue precisely the same methods of experiment, analysis, comparison, and differentiation as are recognized to be necessary to establish any scientific generalization. These methods are not reducible to a precise mathematical form. . . . But that is no reason for ignoring them, or for pretending that the calculation of a probability which takes into account nothing whatever except the numbers of instances, is a rational proceeding. . . . Generally speaking, therefore, I think that the business of statistical technique ought to be regarded as strictly limited to preparing the numerical aspects of our material in an intelligible form, so as to be ready for the application of the usual inductive methods. Statistical technique tells us how to 'count the cases' when we are presented with complex material. It must not proceed also, except in the exceptional case where our evidence furnishes us from the outset with data of a particular kind, to turn its results into probabilities; not, at any rate, if we mean by probability a measure of rational belief." (pp. 391–392.)

For the purposes of drawing inferences from economic statistics, in which the data are not of the "particular kind" indicated by Keynes, the theory of numerical probability gives no assistance. In particular, reasonable forecasts of economic developments can only be made by application of the usual methods of argument. To be sure, the conclusions of such arguments are expressed, not as certainties, but as probabilities. They are not, however, numerical probabilities. It is obviously impossible to state in terms of numerical probability a forecast or an inference based upon both qualitative and quantitative evidence; and even if all the evidence were quantitative, we have seen that it does not express a numerical measure of rational belief for the future. So when we say that "the conclusions of the social scientist are expressed in terms of probabilities" we merely mean that his conclusions do not have the certainty of those of the natural scientist. The probabilities of the economic statistician are not the numerical probabilities which arise from the application of the theorems of Bernoulli and Bayes; they are, rather, non-numerical statements of the conclusions of inductive arguments.

POPULATION GROWTH AND THE VOLUME OF BUILDING

By Willford I. King[1]

In comparing different times and places with respect to the volume of building, the customary method has been to put the building figures on a per capita basis. Is this a logical procedure? It seems rather evident that, in a stationary state, construction work would proceed at a much less rapid pace than it does in our cities at the present time. It is a matter of common observation that in sections of the world in which population is not increasing, new buildings are relatively rare; while, in a rapidly growing community, new structures are seen springing up on all sides. It appears, then, to be far more logical to compare the volume of building with the *growth* in population than with the entire population.

However, in a progressive and prosperous section, there would be considerable new building even if the population remained constant. As the inhabitants became more prosperous, they would demand a better grade of homes than they had before been occupying. Inventions would also cause much new building to house new machines and afford greater conveniences to the inhabitants. Aesthetic reasons would call for the replacement of obsolete buildings with more artistic and hence, as a rule, more expensive structures. In the United States today, this process is steadily going on at a rather rapid rate.

It is, then, clear that buildings are being constructed all the time for both the old and the new population. The extent of building activity in different times and places can, however, be compared satisfactorily only through the use of a composite index combining the two factors—namely, old population and population growth.

The correctness of such an index depends largely upon the accuracy of the weights used. It is, then, highly desirable to know the relative amount of building demanded for each person added to the population as compared to that needed each year for each of the old inhabitants.

The end sought can best be expressed algebraically.

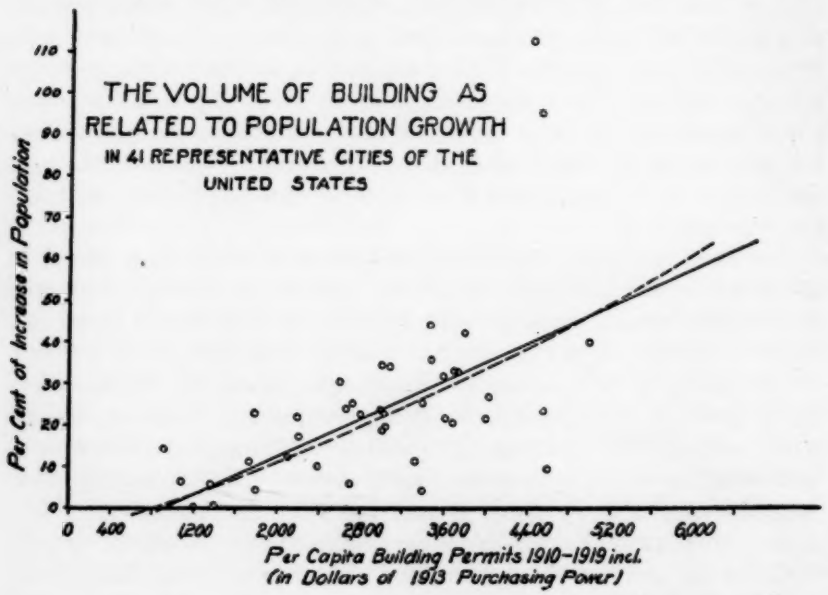
If o = the average population during the decade,

x = the average amount of building constructed during the decade for each of the old inhabitants,

[1] Statistical Work by Maurice Leven.

y = the average volume of construction per person for the added part of the population during the decade,
g = the growth in population during the decade,
then the new building during a decade in any given city necessarily equals $ox + gy$.

Clearly enough, the ratio of y to x may differ in different communities, but there must be an average value of $\frac{y}{x}$ for the country as a whole. An effort is here made to determine this average ratio through a study of the comparative volumes of building permits issued in the leading cities of the United States between the last two Census periods —that is, during the decade 1910 to 1919 inclusive.



We know the increase in population in each city during the decade. The average population has been assumed to be the mean of the two Census counts. The total volume of building permits in each city during the decade has been divided by this mean population to arrive at the value of construction per capita, in the ten-year period. The results are recorded in Table I.

If the population of any given city in 1910 is called 100, then its population in 1920 is 100, plus its percentage of growth. Its average population for the decade, however, is 100 plus half its percentage of growth. If, then, a city's population increased 20 per cent during the decade, the average population, designated by o, would be 110.

TABLE I

PER CAPITA BUILDING CONSTRUCTION AND PER CENT OF INCREASE IN POPULATION IN 41 SPECIFIED CITIES, DECADE OF 1910–20

	Total value of building permits 1910–19 inclusive	Average population 1910–20	Building permits per capita	Per cent of increase in population 1910–20
	Thousands	Thousands		
Boston	$240,164	717	3,349	11.6
Buffalo	122,680	388	3,162	19.6
Cambridge	37,406	107	3,495	4.6
Chicago	654,596	2,443	2,679	23.6
Cincinnati	91,856	382	2,404	10.4
Cleveland	259,002	678	3,820	42.1
Columbus	52,554	209	2,514	30.6
Denver	49,576	135	3,672	20.2
Detroit	324,851	728	4,462	113.3
Fall River, Mass.	15,794	120	1,316	1.0
Grand Rapids	28,028	125	2,242	22.3
Hartford	60,348	118	5,114	39.6
Indianapolis	83,465	274	3,046	34.5
Jersey City	49,367	283	1,744	11.3
Kansas City, Kans.	11,184	61	1,833	22.9
Kansas City, Mo.	107,368	286	3,754	30.6
Los Angeles	205,822	210	9,801	80.7
Louisville, Ky.	40,023	229	1,747	4.9
Lowell, Mass.	14,973	109	1,373	6.1
Memphis	43,766	146	2,997	23.8
Milwaukee	122,177	435	2,808	22.3
Minneapolis	141,631	341	4,153	26.3
Newark, N. J.	115,361	381	3,028	19.3
New Bedford, Mass.	37,071	109	3,401	25.4
New Haven	54,097	148	3,655	21.7
New Orleans	33,886	363	933	14.2
Oakland, Calif.	63,496	183	3,469	44.0
Philadelphia	381,920	1,686	2,265	17.7
Portland, Ore.	106,809	233	4,584	24.6
Providence	63,442	231	2,746	5.9
Richmond, Va.	46,215	149	3,101	34.5
Rochester	86,701	257	3,373	35.6
St. Joseph	9,260	77	1,202	0.7
St. Louis	153,417	730	2,101	12.5
St. Paul	103,916	224	4,639	9.3
San Francisco	185,525	462	4,015	21.5
Scranton	14,926	136	1,097	6.1
Seattle	103,238	276	3,740	32.9
Syracuse	42,693	154	2,772	25.1
Washington, D. C.	137,797	384	3,588	32.2
Worcester	49,723	163	3,050	23.1

The increase in population, or g, would, however, be 20, and the total building requirements would equal $110x+20y$.

In the accompanying chart, the per cent of growth in the population of each city has been plotted against the corresponding per capita value of building permits issued during the decade. The arrangement of the dots representing the various cities seems to leave little doubt that there is a marked degree of correlation between the two variables. A first approximation to the law of the data was made by drawing the solid straight line shown in the chart.

When, however, we try to use this line for forming a series of equations based upon the per cent of growth, the average population, and the product of the per capita building permits and the average population, we find that the ratio of y to x varies somewhat. To make $\frac{y}{x}$ a

constant, the line must curve. Now it may well be that, in reality, $\frac{y}{x}$ is not exactly a constant; but, by adjusting the values, a curve has been arrived at which makes the ratio constant and at the same time appears to fit the data fairly well.

Typical equations based upon the curve are shown below. It will be observed that these equations differ from each other by $10x + 20y$ and that this difference must be a constant if the ratio of y to x does not vary.

TABLE II

Ordinate Increase in population (Per cent)	Index of average population		Index of average population	Abscissa Per capita building permits
0	100x	$+ 0y = 80{,}000 =$	100	$\times$ 800
20	110x	$+20y = 309{,}960 =$	110	$\times$2,818
40	120x	$+40y = 539{,}920 =$	120	$\times$4,499
60	130x	$+60y = 769{,}880 =$	130	$\times$5,922

Note that the curve is produced by plotting the items in the first and last columns against each other.

The whole process is empirical but the result—a curve purporting to represent the law of the data—does not appear unreasonable. The ratio given by the equation is

$$\frac{y}{x} = \frac{11{,}098}{800} = 13.87.$$

The next step is to estimate the volume of building for the United States as a whole during the decade, convert the figures to dollars of 1913 purchasing power, and utilize the results as a basis from which to derive an estimate of the actual values of x and y for the entire United States.

The estimate of the volume of building is based primarily upon the F. W. Dodge figures for years since 1918 and upon building permit figures in earlier years. Adjustments have been made throughout to allow for a roughly estimated volume of non-reported building. The figures used are presented in Table III.

These figures have been applied to the entire population of the Continental United States. The average population used is the mean of the population estimates made by the National Bureau of Economic Research for each January 1 and July 1 during the decade. The derived results follow:

TABLE III

ESTIMATED VOLUME OF BUILDING IN THE CONTINENTAL UNITED STATES

Year	Estimated value of construction in current dollars (Millions)	Index of construction costs	Value of construction work measured in 1913 dollars (Millions)
1910	2,905	.964	3,014
1911	2,829	.970	2,917
1912	3,009	.981	3,068
1913	2,805	1.000	2,805
1914	2,606	.968	2,693
1915	2,651	.984	2,694
1916	3,275	1.168	2,804
1917	2,754	1.440	1,912
1918	2,349	1.604	1,464
1920	3,893	1.896	2,053
Total	29,076		25,424

Population of the United States January 1, 1920	105,710,000
Population of the United States January 1, 1910	91,530,000
Increase during decade	14,180,000
Average population during decade	99,209,000
14,180,000 × 13.87	196,676,000
Total units of population in terms of construction per old inhabitant in a decade	295,885,000
Total building during the decade in 1913 dollars	$25,424,687,000
Building per old inhabitant during the decade; valued in 1913 dollars	$\frac{\$25,424,687,000}{295,885,000} = \$85.9$
Per capita building per year for old population in 1913 dollars	$8.59
Building per new inhabitant in 1913 dollars	$1,191

These figures can readily be converted to present prices by multiplying them by the current index of construction costs.

The results are presented here not with any thought that they furnish a final and accurate solution of the problem, but rather in the belief that they point the way to more careful investigations which will be possible when more accurate data become available, and that they show in a very rough way the actual ratio that prevailed during the decade 1910–20, between the per capita volumes of construction for old and new inhabitants respectively.

THE THEORY OF QUADRATURE IN ECONOMICS

By Karl G. Karsten

In 1908 Mr. Charles N. Edge invented a method of applying the fundamental concepts of the calculus to the treatment of economic data. For fifteen years thereafter he studied this method and out of it he has developed a theory which he calls the "Quadrature Theory" of economics. The quadrature theory states that *the time-lags between cycles in various economic phenomena are functions of the periods of these cycles. The relation between economic forces is such that these forces are in some phase or another always in quadrature with each other.*

Two forces are said by Mr. Edge to be in quadrature when they trace curves such that the fluctuations of one of the curves correspond to the fluctuations of a curve of the integration or cumulation of the data of the other curve. When a cause-and-effect relation exists between such two phenomena, the first may then be said to be cumulatively affected by the second. The small finite changes which take place in the first force are proportional to the data of the second, and vice versa, the integration or summation of the data of the second force yields a series of cumulatives which are proportional to the data of the first force. In the calculus such relations are familiar in the form of integrals and derivatives, and although these functions are purely mathematical, they are useful to describe the behavior of related forces in the physical sciences. It is the quadrature theory that economic data or statistics betray the same relationships when similarly treated, and that when this is the case, the economic forces or phenomena measured by the statistics may be said to be in quadrature and a real relation is strongly suggested.

In the application of the theory to economic statistics, owing to the peculiar nature of the data, it is necessary to use a special technique which may be called the quadrature method. The quadrature method consists of the use of the methods of the calculus of finite differences. The infinitesimal calculus is concerned with infinitesimal differences, and yields what are known as the integral and differential functions. When finite differences are used, the summation or integration yields a cumulative series and the inverse process yields a series of differences. The data of economics are available in weekly, monthly, or other regular time-intervals, and therefore lead to cumulatives in the place

of integrals and differences in the place of derivatives. When the time-intervals are small and regular, the practical distinction between the alternative processes disappears. The method of the calculus of finite differences yields a close approximation to the results which would be obtained by the methods of the infinitesimal calculus if it were practical to apply them. The quadrature method therefore applies cumulation to obtain a series representing the integral function, and differencing to obtain a series of data representing the derivative.[1]

That economic forces usually fluctuate or pulsate, so that the curves of economic statistics are periodic or cyclic, is a fact with which everyone is familiar. Sometimes the waves are very smooth, sometimes they seem cut up with short minor waves, presenting a very choppy appearance. Sometimes the primary or long-time waves are fairly regular, reminding one of sine curves; sometimes they are very irregular, large waves and small waves, short and long, all following close upon each other. But whatever their form, these wave-like irregularities are present in almost all economic data. Obviously a theory by which we would correlate the integrations or cumulations of any data with the data of such periodic curves must be a theory according to which periodic integrals can be produced. Cumulations of wholly positive data will constantly increase and trace rising curves only. Cumulations of wholly negative data will trace constantly falling curves. Only the cumulations of data which contain both positive and negative items trace curves which will both rise and fall, rising as positive items are added and falling again as negative items are added. And only when the positive and negative items balance, giving equal areas above and below the zero-line, will the curve of the integral fluctuate around a horizontal trend, that is, begin and end at the same level.

The quadrature theory therefore postulates trends, "normals" or central positions, in all economic phenomena. The concept of the proper trend is moreover clarified and a definite test is afforded for its

[1] The following definitions are taken from the Century Dictionary:

"Quadrature," in electricity, phase difference of 90° or one quarter-period.

"In quadrature," in electricity, having a phase difference of 90°.

"Method of quadratures," the approximate integration of an expression between given numerical limits by the summation of parts in each of which the difference between the limits is so small that the integral is practically equal to that of some integrable expression.

"Period," in physics, the time of one complete oscillation or cycle of a periodic motion; the reciprocal of the frequency of a periodic motion.

"In phase," synchronous as to phase, said of two or more periodic motions or cyclic processes of whatever nature.

"Integral" in mathematics, the result of integration, or of the operation inverse to differentiation. An integral is either (a) the integral of a quantity or (b) the integral of an equation.

The now unused, if not exploded, "method of variate differences" is not to be confused with the "quadrature method," as it was directed to the determination of the correlation of the successive difference series upon the same basis as the constant-time-lag method, namely, that the two original series are taken as being in the same phases of their cycles.

determination. Present methods of handling trends and "normal lines" involve the use of horizontal averages, linear trends, exponential and parabolic curves and the like; indeed it may be that for certain purposes another irregular periodic curve, such as of related economic data, can logically be the trend desired. But present methods give no clear tests of the virtues of these. The quadrature theory makes it possible quite precisely to select the best of several good trends. For in the quadrature theory, the correct trend is clearly defined as that central position about which the deviations of the statistics may be regarded as significant positive and negative forces, the data of which, when cumulated, really correlate well with such other data as they logically should so correlate with.

The quadrature theory would seem to hold out a promise of reconciling pure theory and actual facts, in the matter of the length of cycles in economic data. For it has long been observed that business cycles are not of equal duration or violence. Mathematically stated, they have unequal periods and amplitudes. It has become clear that the Lord has not decreed that in our civilization the phases of economic cycles are functions of equal time periods, such as eight years or three and a third years, or any other functions of the rotation of the earth about its sun. By the quadrature theory, it seems possible to interpret the various phases of the economic cycle as entirely orderly and in accordance with theory, and yet wholly irregular in point of time.

It is almost always observed that the time-lag varies in length in the progress of any two phenomena. Such a statement as that "curve B follows curve A by from two to ten months" is typical. The quadrature theory is that *the time-lags between the cycles of various economic phenomena are constant functions of the periods of the cycles.* If we take the familiar case of a regular sine curve and its integrals, we find that the first integral is a quarter-period behind it, the second integral is a half-period behind, the third three-quarters behind, and the fourth, being one full period behind, retraces it. In this case the lags are constant in length. But the cycles in economics are not pure sine curves and their integrals (or cumulatives) must be lagged behind them by various time-lengths, though still constant functions. Thus the first integral (or cumulative) is what may be called a quarter-period behind the integrand, having a minimum or maximum as the latter crosses the zero-line, although this "quarter-phase" may contain two months or ten months. In short, the quadrature theory would seem to offer a new type of time-lag in economics, which is irregular in time-length and yet wholly precise, measurable, and predictable.

Present methods of correlating economic data involve the use of the

constant-time-lag method. The object seems to be to find the exact length of time by which one set of data may be lagged throughout its entire length to yield the highest correlation with another set of data. In so far as there is a theory behind this method, it is that some significance attaches to the length of the time-lag which yields the best correlation. It would assume, for example, in the case of idle freight cars and interest rates, that the effects of a car-shortage are not felt in the money market for a period of from nine to twelve months afterward. Under modern conditions of communication, the sensitive structure of credit in the civilized world is no such laggardly thing; instead we may expect effects to be instantaneous, if not frequently discounted in advance. Even if the cycles in these two series were entirely regular and the time-lag between them constant, the theory of constant time-lags would fail to account for the lag. Thus it may be that the time-lag method of correlation is, in many instances, without a sound under-structure of logic and reason. It may be that the quadrature theory, with its "quadrature-lag" of the cumulation of the effects of one series upon another, will provide a rational basis, a sound raison d'être, for these lags, be they regular or irregular.

The lags produced by the quadrature theory are not limited to the exact quarter-, half-, and three-quarter-periods of pure integrals. Other fractions of the cycle may occur through the combination of different phases as causative forces. Thus purchasers everywhere watch not only the present level of prices, but also the direction in which these prices are changing and are expected to change. Here we have as a generating force a composite of the price-curve and its derivative, and so may expect a curve of purchasers' stocks (if consumption be constant), showing the accumulated effects of purchasing, to vary as the mean of the price-curve and its integral, and to be lagged behind the price-curve by a lag of one-eighth of the period of the cycle in prices. To such a mean between a curve and its first integral there has been given for the purposes of this work the name of the "first semi-integral." With changes in the technique of buyers, it may even be possible that the weights given to the consideration of the price-level and the consideration of its direction of change, will alter from time to time, so that at one time the curve of cumulated effects of buying will be nearer to the price-curve and at another time nearer to the integral of the price-curve. With such varying relations in the quadrature of two curves, the quadrature theory is also concerned.[1]

[1] It may be added that Mr. Edge also makes some use of moving or progressive totals or averages placing them at the ends of their periods. Unlike the definite integral between the points T_0 and T_n, the moving annual total is an integration between the points T_{n-12} and T_n. It is, of course, intended for the elimination of seasonal cycles, and yields a relatively smooth line which approaches the trend as

In these respects the quadrature theory would seem to be of value in reconciling pure theory with observed fact. In the fixing of trends it would seem to provide a useful analytical instrument. The purpose of economic research is the determination of the nature of the inter-relations and inter-actions between economic forces, to the end that the future course of economic events may be forecasted. In the wide variety of such inter-actions which it premises, and in the definiteness of the mathematical statement of these, the theory would seem to offer a means for further progress in such research.

Attention may be called to an important corollary of the quadrature theory, that the precision with which the integral of a series can be calculated is relatively far greater than the precision with which its derivative can be calculated. This will be apparent from an examination of the mathematics of the calculus of finite differences.[1] This fact is of great importance in forecasting. Thus if it be shown that the price-changes, that is, the derivative of prices, trace a curve parallel to the volume of business, that fact is, of itself, of little value in forecasting the volume of business, even if we have a fairly reliable prediction of prices, because the probable error of the price prediction becomes enormous in the curve of its derivative. But it we find a means to predict the volume of business with only fair reliability, we shall be able to use it to predict the price-level within a very small

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the period of the total approaches the period of the major cycle. Mr. Edge thinks it valuable in that it affords a smoothed line which, when plotted at the end of its period, is often near to a smoothed line through the first semi-integral, but is always up-to-date; whereas the smoothed line through the semi-integral cannot be definitely determined until sufficient succeeding data have been learned.

[1] The following short proof may be used: The problem is concerned with the erratic or insignificant fluctuations of the statistics of economic data, whether these fluctuations be of the nature of true "errors" or not. Every item in a series of data is itself an aggregate, or result of observations and as such is liable to error. Call the probable errors of the individual items each "*p.e.*" and assume that for all items in a given series the size of "*p.e.*" is the same, a certain constant quantity. In the ratio $\frac{p.e.}{\sigma}$ we have a measure of the relative importance of these probable errors as compared with the cyclic fluctuations. Using the subscripts $_o$, $_d$, and $_i$, to indicate these properties of items in the original or integrand, derivative, and integral series, respectively, the problem is to find the relations between $\frac{p.e._o}{\sigma_o}$, $\frac{p.e._d}{\sigma_d}$, and $\frac{p.e._i}{\sigma_i}$, or, in other words, to find the last two in terms of the first and of each other. $P.e._d$ may easily be shown to be equal to $p.e._o\sqrt{2}$, as it is the probable error of the difference of two items. When the derivative traces a curve which is periodic like that of the integrand, it has cycles of approximately the same length, and if n be the number of observations per cycle, the amplitude of the integrand or original series, being the integration of the derivative or difference series, will be about $\frac{1}{4}n$ times the amplitude of the difference series and so, σ_o^* will be about $\frac{1}{4}n\sigma_d$. Therefore $\frac{p.e._d}{\sigma_d}$ is approximately $\frac{n\sqrt{2}}{4}\frac{p.e._o}{\sigma_o}$. As n contains around 40 items (for monthly, data) in the average cycle and is rarely below 20, the relative importance or errors in the original data is increased at least eight-fold and in general sixteen-fold in the derivative series, and conversely is diminished to about one-sixteenth in the integral, the error of items in the second integral series being very small.

margin of error, because the probable error becomes much smaller after cumulation. If we find a third series, the second integral of which correlates with prices, the probable error of a forecast disappears altogether for practical purposes. And if no other means of forecasting were possible, it would still be of practical value to project the smoothed curve of the integrand through the curve of the integral, merely estimating for this purpose the most probable length of the time-lag during the most recent portions of the cycle. These applications of the quadrature theory will be appreciated by those who are seeking to forecast phases of the business cycle.

In October, 1922, a bureau was founded by Mr. Edge, called the Quadrature Bureau, for the purpose of investigating economic cycles by the quadrature method and so far as possible forecasting the future course of these cycles. During 1923 research was devoted in turn to interest rates, the prices of industrial stocks, the prices of bonds, and the course of general business activity. For each of these, correlations were found with the integrals or other quadrature functions of entirely distinct, although logically related data—thus the forecast of interest rates is based upon the surplus or shortage of freight cars. Some promising work was also done upon forecasts of New York factory wages and of the Bureau of Labor Statistics index of wholesale prices. In all these cases a high correlation has first been sought between the data of the thing to be predicted (or criterion) and the integrals or similar functions of the data of wholly separate economic forces. After this correlation has been found the forecasting for from six to eight months in the future is accomplished through various processes with which we are not here concerned. The major task is the selection, through research and experiment, of data which, after integration, yield high correlations with the criteria over long periods of time (that is, for as long as the data are available and for at least several years). By permission of the Quadrature Bureau, there are herewith presented data and charts of the series correlating with interest rates, and charts of the series correlating with the curve of general business activity. As the forecasts for the next year are the property of the Bureau and its clients and are not essential to this discussion, they have been omitted.

The procedure outlined below is that developed by the present writer, as combining a high degree of accuracy with relatively simple steps. It is not claimed that it is the only method. Somewhat finer results can be assured by fitting the two curves (the cumulative and the criterion) by the method of least squares. Mr. Edge, on the other hand, uses much simpler methods than the one here outlined and

obtains as useful results for practical purposes. It is always helpful to plot curves of every series (original or derived, such as cumulatives) promptly and to learn to visualize from curves of the original data the corresponding curves of integrals. In the hands of an expert little more than good curves is necessary to detect the possibilities of correlation of integrals and determine the methods of analysis and indeed often the required constants of fit to be used in the further steps by the quadrature method.

The following procedure is given to show the general steps by which any investigation can be carried out, although it has been particularized for the case of freight-car shortage and interest rates (on 60-90-day commercial paper): (1) Change the signs of the data of freight-car surplus or shortage (as a shortage indicates activity and a surplus inactivity); (2) compute the deviations of the series from the mean (thereby taking the mean as the empirical trend); (3) cumulate the last data, beginning naturally with zero as the item immediately preceding the first cumulative figure (that is, with zero as the "added constant of integration"); (4) convert the cumulatives into deviations from their mean (thereby correcting the added constant to about what it should be); (5) amplify the last data by the ratio of the average deviations of interest rates to the average deviations (standard deviations are not worth the additional labor) of the last data (thereby securing equal amplitudes for the two curves to be correlated); (6) increase the amplified items by the means of the interest rate series (thereby positioning the two curves at the same height upon a chart). The above steps adjust the integral of car-shortages so as to fit the interests rates in the manner of a first approximation.

In order to determine the trend, if any, of the car-shortage data, and refine the fit of its integral to the interest rates, the following additional steps may be taken: (7) Find the residuals of the car-shortage integral from the interest rates (by subtraction); (8) divide these residuals by the same amplifying factor used as a multiplier in step 5 above; (9) take the successive differences of the last data (thereby securing a series representing the error in the assumed horizontal mean trend); (10) fit to these errors any desired mathematical trend by the usual mathematical methods (least squares and the like) or any other series such as the data of other related economic series (just as the general price-level may be the best trend to use for the prices of any particular commodity); (11) check the proposed trend by cumulating its figures and comparing with the de-amplified residuals of step 8 above (and find the mean of this cumulative series to use as a correction of the added constant of integration). The above steps provide the

refinements with which to improve the correlation between the car-shortage integral and the interest rates, that is, the correlation with the least probable error. When the first approximation has yielded a poor fit, as shown by a large trend in the residuals, it is often advisable to repeat all the steps, using the corrections which have been determined. Thus the mean which was used as an empirical trend in step 2 should be corrected by the trend adopted in step 10, and the constant of integration as adopted in step 4 should be corrected by the correction found in step 11; and the average deviations used in step 5 should be recomputed. It is also sometimes well to take a linear or simple parabolic trend out of the criterion in the first place when this is markedly evident. As a second degree parabola is the result of cumulating a linear trend, it is evident that correcting for a second degree parabola in the integral series is the same as correcting for a linear trend in the integrand. Moreover, it may sometimes be best to use the logarithms of a series in fitting, especially in data of prices and interest rates.

Having determined in this way the constants of the trend, the amplitude, and the added constant of integration, the correlation can be produced in revised short-cut order in four simple steps: (1) Write the series of figures representing the ordinates of trend (using the series obtained by step 10 above increased by the mean eliminated in step 2 above); (2) compute the deviations of the freight-car shortage from the last mentioned data of trend (writing shortages as positive and surplus as negative figures); (3) cumulate these deviations from the correct "added constant of integration" (a figure which is obtained by dividing the mean of the interest rates found in step 6 above by the amplifying factor used in step 5 above and diminishing the quotient by the sum of the empirical constant of integration used in step 4 above and the correction for this found in step 11 above); (4) amplify the cumulative series by the amplifying factor found in step 5 above. In these steps the figures of freight-car shortage or suplus will be adjusted to correlate best with the figures of interest rates. It is these four steps that form the permanent records, and are here shown in the tables and charts. In the present case, as no trend was found in the interest rates, the mean is the trend; and the first step, calculating the ordinates of trend at each month, is omitted.

It will be seen in the chart that the correlation is good. For the years 1915–23 inclusive, the coefficient of correlation between the two series shown in the table and plotted on the chart, is +.914. It is legitimate, however, to disregard the poor fit during the nine months, March to November, 1918, during which time the interest rates were

arbitrarily prevented from rising above 6 per cent, as under such circumstances the forecasting curve can only tell where the interest rates would have gone if they had not been fixed; for the remainder of the

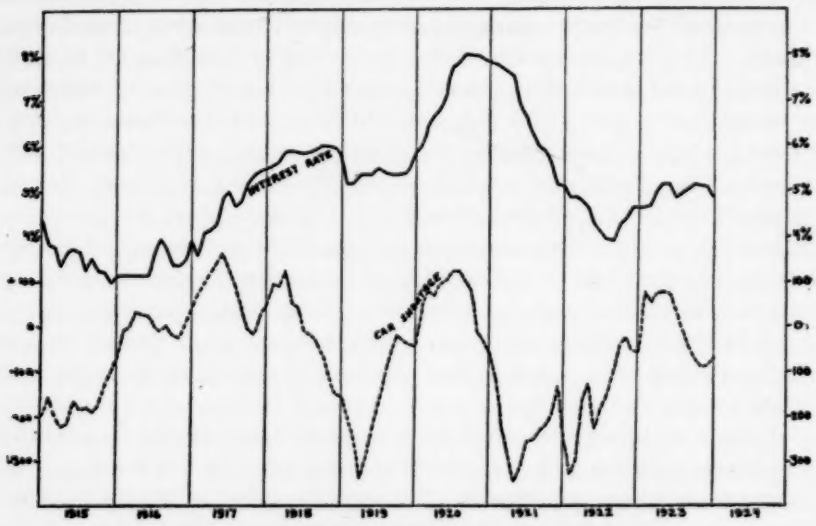


CHART I. INTEREST RATES AND CAR-SHORTAGE

The full line is the curve of interest rates on 60–90 day commercial paper (choice double-name) by months from 1915–23. The points plotted are the monthly averages of the high and low weekly quotations published by the *Commercial and Financial Chronicle*.

The broken line is the curve of net freight-car shortage or surplus on the first of each month during the same period. Shortage is plotted upward as positive values and surplus is plotted downward as negative values. (The chart was drawn by Mr. Robert W. Edge of the Quadrature B.)

The coefficient of correlation between the two series as they are plotted is +.0234. If the data of car-shortages be lagged four months the coefficient is +.239; if a lag of eight months be used, it is +.402, and if a lag of one year be used, it is +.337.

nine years the coefficient of correlation is +.923. It may also be observed that interest rates tend to change in geometric progression rather than arithmetically. If the car-shortage cumulatives are fitted to the logarithms of interest rates and then their anti-logarithms taken as the forecasting or correlating series, the correlation is still better. The coefficient of correlation between the present forecasting series and the logarithms of interest rates is, for the entire series, +.926, and for the series when the rate was not fixed, +.930. Lastly, it may be pointed out that the integral curves showing the cumulative effects of one economic force upon another trace smooth lines of what may be called the underlying tendency, but do not usually take account of purely seasonal variations in the resultant phenomenon. Thus the real test of the fit is in the correlation between the cumulative of car-shortages and the moving annual average of the interest rates. For the nine years, apart from the time when the interest rate was fixed,

the coefficient of correlation between the cumulatives and the logarithms of the moving annual average of interest rates is +.950.

Car-shortage data are available only from 1907 on, and are in very poor shape, statistically, for several years thereafter. It was not until the war that the figures were compiled with such care as is used nowadays. For many years the estimates were issued as of uneven dates in the month, and it is necessary to interpolate to estimate conditions at the end of each month. The reliability of the car-shortage data is in general much less in the period of 1907–14 than in the later period. Interest rates also fluctuated much more violently prior to 1915, and a large seasonal cycle was present which has since disappeared. Indeed, it is possible that the effects of the Federal Reserve system, which are known to have eliminated the seasonal cycles, are so great as to make interest rates before 1915 not comparable with the same rates after 1915, so that they should not be compared with correlated data on the same basis for the two periods. If this is the case, it would justify the use of different amplitude factors and different constants of integration and of trend, for correlated data in the earlier period. The Quadrature Bureau has, however, not made any more or less arbitrary assumptions, but has continued the cumulated series of car-shortage back to 1907 treating the entire period 1907–23 as a unit. The two curves do not have as good a fit during this earlier period, a condition which is to be expected. During 1907 to 1914 the coefficient of correlation between the adjusted cumulatives and the logarithms of the moving annual average of interest rates is +.856.

The significant facts which may be gleaned from this example are as follows: (1) Whenever the freight-car surplus amounts to more than 97,000 cars, net, the rates on 60–90-day commercial paper fall, and whenever there is a shortage, or the surplus amounts to less than 97,000 cars, the interest rates rise; (2) the change in the interest rates amounts to one-eighth of a per cent per month for every 144,000 cars; so that during a surplus of 241,000 cars (that is, 144,000 more than 97,000) the interest rates fall one-eighth of a per cent per month; during a surplus of 385,000 cars (that is, 288,000 more than 97,000), the interest rates fall one-quarter of a per cent per month; and vice versa, during a shortage of 47,000 cars (that is, 144,000 less than 97,000 surplus), the interest rates rise an eighth of a per cent per month—and so on; (3) the interest rates for the future may be forecasted as far ahead as the condition of car-shortage or surplusage may be predicted. Thus if, let us say, a surplus is expected of 100,000 cars on January first, 200,000 cars on February first, and 300,000 cars on March first, the rates on commercial paper may be expected to be 4.90, 4.81, and 4.63

during these months.[1] If the probable errors of the car-surplus estimates be respectively ±50,000 cars, ±100,000 cars, and ±200,000 cars, the probable error of the interest rate estimates will be ±.04, ±.1 and ±.2 respectively. It has already been observed that the probable error is relatively much smaller in the integral than in the integrand series. In this discussion of forecasts, the amounts quoted do not represent considered estimates by the bureau, but figures selected at random to illustrate the greater precision of the integral series.

It is proper to compare the correlation which has been secured by the quadrature method, with the correlation which would be reached by the time-lag method, as this method would be the alternative procedure in analysis. By the quadrature method a correlation coefficient of .914 was obtained for the entire series from 1915–23 inclusive (.950 when logarithms of the moving average are used and nine months excluded). A glance at the chart, however, will show that by the constant-time-lag method the fit is so poor that the investigation would not be attempted. The coefficients of correlation for a lag of one-half month, +.0234, for a lag of 4½ months, +.239, for a lag of 8½ months, +.402, and for a lag of 12½ months, +.337. The coefficients for intervening months have not been calculated, as it is obvious from the chart that no great improvement is possible. It is not necessary to add that this is one of those cases in which even the best of correlations with any considerable time-lag would have been difficult to interpret, owing to the lack of any logical reason for such a delay between the occurrence of cause and the occurrence of effect, or between the occurrence of related effects.

It will be seen that the quadrature method is directed toward the determination of relations between economic phenomena, the nature of these relations being premised by the quadrature theory. The

[1] It will be noticed that in this case the forecasts have been appended to the last value of the interest rate series, by producing a parallel to the forecasts which would be extended from the cumulative series. For while the shape and direction of the integral is always significant, its position upon a chart is entirely the result of the trend which has been eliminated or added in the process of integration. Sometimes the integral can be fitted so precisely to the criterion as to give confidence also in the position which has been assigned to it, but where there is not full trust in this, it is always of practical value to extend the forecasts from the actual position of the criterion, or of a smoothed line through the criterion. In the case of surplus cars, as Mr. Snyder has pointed out to the writer, a variation in the trend line occurs due to the replacements of railway equipment, and unless this has been corrected for, the cumulative series must be expected to drift slowly to one side or the other of the desired position, although still retaining a general parallelism and hence a significance in its direction. Indeed, it would be a more precise, though somewhat less clear, method of presentation, to show the integral curve as a shaded zone, letting the width of the zone be the probable error of the fit during the surrounding years. For, as will be seen, the obstacles to a good fit are not alone in errors of the statistics of the integrand, but also in errors of the statistics of the criterion, or thing to be fitted, using the word error in a technical sense. Interest rates, for example, are different in different cities, and vary in the same city through the month. In the present study, the arithmetic mean between the highest and lowest quotations published weekly by the *Commercial and Financial Chronicle* has been used.

theory is a statement of the principle underlying such relations as the method detects, and the method is intended for the proof or disproof of

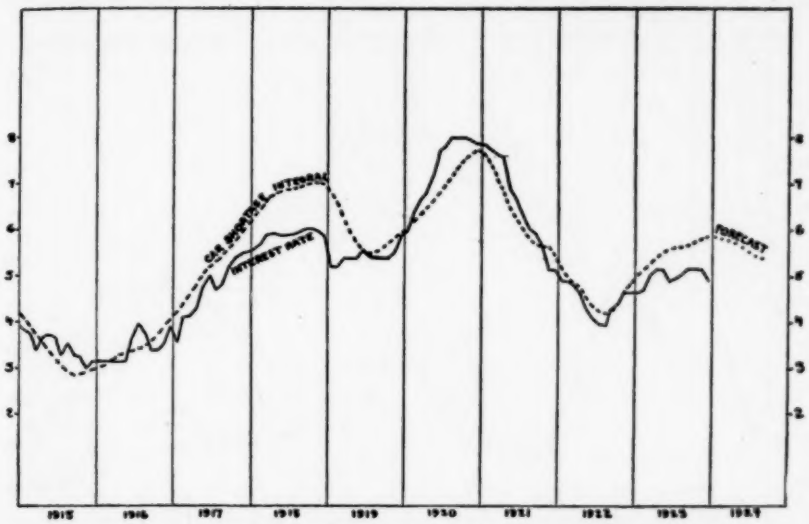


CHART II. INTEREST RATES AND INTEGRAL OF CAR-SHORTAGE

The full line shows, as in Chart I, the curve of interest rates. The broken line is the curve of the integral (or cumulative) of the deviations of car-shortages from the mean of car-shortages. Dots projected across 1924 show the probable future course of this curve during the first eight months of 1924, as forecasted by the Quadrature Bureau, 402 Madison Ave., New York City. (The chart was drawn by Mr. Robert W. Edge of the Quadrature Bureau.)

The coefficient of correlation between the two series is +.914. Between the integral of car-shortages, and the logarithms of the moving annual average of interest rates, when the period of restricted rates during 1918 is disregarded, the coefficient of correlation is +.950.

such relations when they have been suspected. In the case which has been used as an example, the relation has proved quite simple between two series of data. The curves of the index of general business conditions, also shown in graphic form in these pages, is an example of more complex work, the index of actual business conditions being a composite of many different items and the correlating curve also being the result of combining several series of data, and subjecting them to the processes of cumulation.

Certain forms of cumulation are so simple as to have had immemorial use. Thus the amount of stock in a warehouse is the cumulative of net warehouse receipts, that is, of the difference between receipts and withdrawals. A bank balance is the cumulative of deposits and withdrawals. Instances of the cumulative in business and economics could be multiplied endlessly; indeed all stock or fund figures in economics, as opposed to stream or flow figures, are cumulatives. But from these it is only a step to think of wholly distinct items which are not them-

selves cumulative, but which vary with these cumulatives, and so we find parallels in daily life to the fundamental relations between distinct economic phenomena, with which the quadrature theory is concerned.

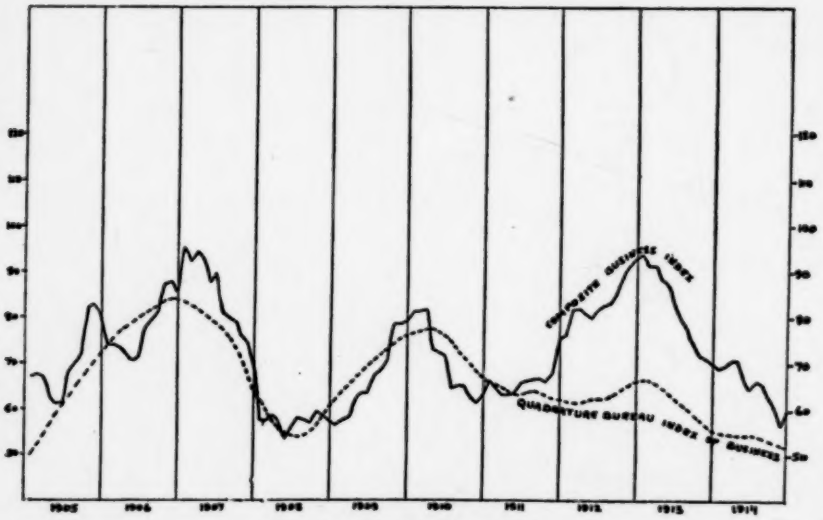


CHART III. CURVES OF

The full line is the curve of a "Composite Index" of general business conditions, combining the following elements; unfilled orders of the U. S. Steel Corporation; after 1906, net freight-car shortage or surplus; and after 1914, employment in New York State factories, and the Federal Reserve Board index of the volume of production in 22 basic industries.

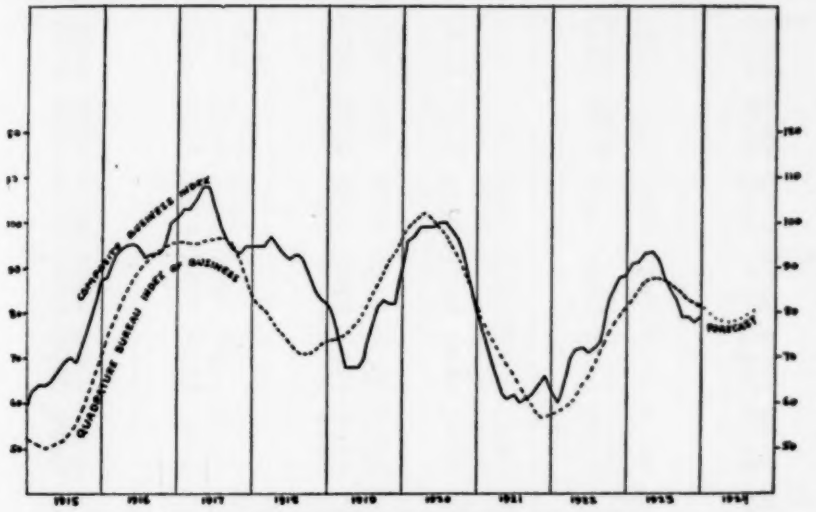
The broken line is the curve of the integral of a series compiled by the Quadrature Bureau to represent fundamental conditions which have cumulative effects upon business conditions. The dots

Or, if we wish, we may find parallels which can be easily visualized in the physical sciences, such as the relation between the flow of water through an aqueduct and the height of water in a reservoir, or the relation between acceleration, speed and distance, the classical illustration of integrals in the text-books. The following comment by Professor Macaulay is to the point:

The quadrature theory of economics claims to be more than a mere statistical or mathematical technique. It claims that the differences and summations which it presents are not mere mathematical abstractions, but correspond to realities in the relations between the phenomena themselves. It claims to be more than any mere Bode's law. In so far as the words have meaning in modern philosophic thought, the theory claims rationality as opposed to mere empiricism.[1]

[1] No discussion of the quadrature theory would be complete without reference to notable work by other investigators pointing toward the same conclusions as Mr. Edge has reached. Early in 1922, Mr. Leonard Ayres published a series of the deviations of interest-rates from bond-yields and showed that when these deviations crossed the zero-line, the prices of industrial stocks reached a maximum or

a minimum. Moreover, in the preceding issue of this JOURNAL, Professor Irving Fisher presented curves of the price-changes and the volume of trade, and using the word derivative for these price-changes, pointed out the correlation between the two. These things seem to indicate that the quadrature theory would sooner or later have been formulated by various investigators. The present writer was first introduced to the principle by Mr. Edge in 1922, and for more than a year thereafter was



GENERAL BUSINESS CONDITIONS

across 1924 show the probable future course of this curve, as forecasted by the Bureau. (The charts have been drawn by Mr. Robert W. Edge of the Quadrature Bureau.)

The fit of the two curves during the period 1912–15 is not so good, indicating that too great an upward linear trend has been eliminated from the data of fundamental conditions, leaving a parabolic trend in the integral. Such errors of trend are easily corrected and have little significance therefore, the important features being the coincidence of highs and lows and the changes in the slope or direction of the curves.

privileged to conduct an independent investigation of it, being inspired to undertake this work by Mr. Carl Snyder. He has been constantly indebted to Professor William F. Ogburn for friendly criticism and advice. The final stimulus to present this statement of the theory has come from Mr. Fred R. Macaulay, who has, moreover, devoted valuable time to the criticism of this statement and has made invaluable suggestions regarding the theory. Professor Wesley C. Mitchell has very kindly read through the manuscript.

	Freight car surplus and shortage (net) (−,000 omitted)	(Same) Deviations from mean (and signs changed)	(Same) Cumulatives from 4,855 as of Dec. 15, 1914	(Same) Amplified by the factor .000,867	Interest rates on 60-90-day commercial paper (mean of high and low)
	As of 1st of month		As of 15th of month		
1915 Jan.	242	−145	4710	4.132	3.84
Feb.	279	−182	4528	3.974	3.75
Mar.	322	−225	4303	3.779	3.38
Apr.	327	−230	4073	3.580	3.66
May	291	−194	3879	3.412	3.72
June	300	−203	3676	3.236	3.65
July	276	−179	3497	3.081	3.25
Aug.	265	−168	3329	2.935	3.53
Sept.	185	− 88	3241	2.859	3.25
Oct.	78	19	3260	2.875	3.22
Nov.	28	69	3329	2.935	2.98
Dec.	37	60	3389	2.987	3.13
1916 Jan.	47	50	3439	3.030	3.13
Feb.	20	77	3516	3.097	3.13
Mar.	− 19	116	3632	3.197	3.13
Apr.	5	92	3724	3.277	3.13
May	33	64	3788	3.332	3.13
June	58	39	3827	3.366	3.63
July	52	45	3872	3.405	3.97
Aug.	10	87	3959	3.480	3.73
Sept.	− 20	117	4076	3.582	3.38
Oct.	− 61	158	4234	3.720	3.38
Nov.	−115	212	4446	3.904	3.50
Dec.	−108	205	4651	4.082	3.91
1917 Jan.	− 62	159	4810	4.220	3.55
Feb.	−110	207	5017	4.400	4.09
Mar.	−130	227	5244	4.597	4.13
Apr.	−145	242	5486	4.807	4.28
May	−149	246	5732	5.020	4.83
June	−107	204	5936	5.197	5.00
July	− 77	174	6110	5.348	4.68
Aug.	− 34	131	6241	5.462	4.81
Sept.	− 35	132	6373	5.576	5.19
Oct.	− 70	167	6540	5.721	5.38
Nov.	−140	237	6877	5.927	5.47
Dec.	−117	214	7091	6.113	5.50
1918 Jan.	− 90	187	7278	6.275	5.58
Feb.	− 98	195	7473	6.454	5.69
Mar.	−138	235	7708	6.657	5.88
Apr.	− 28	125	7833	6.765	5.90
May	12	85	7918	6.839	5.88
June	71	26	7944	6.862	5.88
July	63	34	7978	6.891	5.88
Aug.	56	41	8019	6.927	5.94
Sept.	36	61	8080	6.980	6.00
Oct.	46	51	8131	7.024	6.00
Nov.	92	5	8136	7.028	5.97
Dec.	157	− 60	8076	6.976	5.86
1919 Jan.	296	−199	7877	6.804	5.19
Feb.	354	−257	7620	6.581	5.19
Mar.	473	−376	7244	6.255	5.38
Apr.	426	−329	6915	5.970	5.38
May	366	−269	6646	5.737	5.38
June	285	−188	6458	5.574	5.53
July	188	− 91	6367	5.495	5.42
Aug.	89	8	6375	5.502	5.38
Sept.	− 17	114	6489	5.601	5.38
Oct.	− 52	149	6638	5.730	5.38
Nov.	− 49	146	6784	5.856	5.50
Dec.	7	90	6874	5.934	5.88

	Freight car surplus and shortage (net) (−,000 omitted)	(Same) Deviations from mean (and signs changed)	(Same) Cumulatives from 4,855 as of Dec. 15, 1914	(Same) Amplified by the factor .000,867	Interest rates on 60-90-day commercial paper (mean of high and low)
	As of 1st of month		As of 15th of month		
1920 Jan.	− 26	123	6997	6.041	6.00
Feb.	− 85	182	7179	6.199	6.41
Mar.	− 63	160	7339	6.337	6.68
Apr.	− 81	178	7517	6.491	6.81
May	− 69	166	7683	6.635	7.16
June	−104	201	7884	6.809	7.72
July	−122	219	8103	6.999	7.84
Aug.	−124	221	8324	7.190	8.00
Sept.	−147	244	8568	7.402	8.00
Oct.	− 78	175	8743	7.554	8.00
Nov.	− 52	149	8892	7.683	7.94
Dec.	30	67	8959	7.741	7.88
1921 Jan.	194	− 97	8862	7.657	7.83
Feb.	323	−226	8636	7.461	7.75
Mar.	413	−316	8320	7.188	7.63
Apr.	496	−399	7921	6.842	7.55
May	482	−385	7536	6.509	6.88
June	394	−297	7239	6.252	6.63
July	373	−276	6963	6.013	6.28
Aug.	318	−221	6742	5.821	6.00
Sept.	246	−149	6593	5.692	5.90
Oct.	169	− 72	6521	5.630	5.65
Nov.	69	28	6549	5.654	5.13
Dec.	283	−186	6363	5.493	5.13
1922 Jan.	470	−373	5990	5.170	4.88
Feb.	330	−233	5757	4.968	4.88
Mar.	244	−147	5610	4.840	4.78
Apr.	206	−109	5501	4.747	4.60
May	371	−274	5227	4.510	4.25
June	303	−206	5021	4.332	4.05
July	234	−137	4884	4.213	3.94
Aug.	150	− 53	4831	4.166	3.91
Sept.	12	85	4916	4.240	4.25
Oct.	−124	221	5137	4.431	4.38
Nov.	−175	272	5409	4.667	4.63
Dec.	−127	224	5633	4.861	4.63
1923 Jan.	− 68	165	5798	5.004	4.63
Feb.	− 49	146	5944	5.130	4.69
Mar.	− 69	166	6110	5.274	5.00
Apr.	− 60	157	6267	5.410	5.13
May	− 24	121	6388	5.515	5.12
June	16	81	6469	5.585	4.87
July	52	45	6514	5.624	4.94
Aug.	66	31	6545	5.651	5.02
Sept.	57	40	6585	5.686	5.12
Oct.	26	71	6656	5.748	5.12
Nov.	12	85	6741	5.822	5.10
Dec.	42	55	6796	5.870	4.87

FOUR TYPES OF INDEX NUMBERS OF FARM PRICES

By L. H. Bean and O. C. Stine, *Division of Statistical and Historical Research, Bureau of Agricultural Economics*

It is the purpose of the following statement to present four types of aggregative index numbers of the prices of agricultural products and to consider the type best suited for a particular purpose. The four types to be considered may be briefly described as (a) an index of prices weighted by fixed annual quantities sold, (b) an index of prices with seasonal variations eliminated weighted by fixed annual quantities sold, (c) an index of prices weighted by monthly sales, and (d) an index of prices with seasonal variations eliminated weighted by monthly sales.

In analyzing farm prices, it is especially important to consider seasonal variations in price and in quantities marketed. The current widely used index numbers of wholesale prices [1] have been constructed without reference to seasonal variations in prices and marketings, partly on account of the difficulty of securing satisfactory data as to the marketings of products by months and partly on account of the fact that using a large number of miscellaneous commodities and wholesale prices to some extent obviates the necessity of considering seasonal marketings. The fact is recognized by Sir G. H. Knibbs [2] that "price indexes based upon a constant composite unit are therefore *not strictly* accurate for any particular date though 'on the average' they are accurate." He concludes that "the error, however, in nearly all cases is quite negligible." The errors resulting from the use of constant per annum quantities may be inconsequential in an index based upon a large number of commodities. The effect of seasonal price and quantity movements of some commodities may in this case be offset by seasonal tendencies of other commodities. In an index of farm prices, however, the commodities included are necessarily few in number compared with those in an all-commodity index and the majority of them have marked seasonal characteristics. Here offsetting seasonal variations cannot be relied upon to produce fairly accurate results. Furthermore, in a separate index for a smaller group of farm commodities, such as fruits, vegetables or dairy products, it

[1] Bureau of Labor Statistics All Commodity Index and Fisher's weekly index of 200 commodities.

[2] G. H. Knibbs. *Price Indexes, Their Nature and Limitations, the Technique of Computing Them and Their Application in Ascertaining the Purchasing Power of Money.* 1918. Page 22.

is still more essential that the method of construction make allowance for seasonal changes.

Monthly farm prices are available from August, 1909, to date. The five-year period, August, 1909–July, 1914, is the most nearly "normal" period for which data are available for the purpose of determining the seasonal variation in farm prices. Taking this period as base, seasonal variation in prices may be eliminated from index numbers by dividing the price (or aggregate) of any given month by the average price (or aggregate) of the five corresponding months of the base period, August, 1909–July, 1914.

The average sales for each month of a "normal" year or period may be used as quantity weights in order to allow for normal seasonal variation in the quantities marketed. Sales in the period 1918–22 are used in the following computations:

If we let Pi = Price, given month,
Po = Average base price,
$Pc.m.$ = Base price for corresponding month,
Qa = Quantity per annum, and
$Qc.m.$ = Quantity for corresponding month,

then the four possible types of aggregative index numbers, expressed in formulae are as follows:

$$\text{Type A} = \frac{\text{Sum } (Pi \quad Qa)}{\text{Sum } (Po \quad Qa)}$$

$$\text{Type B} = \frac{\text{Sum } (Pi \quad Qa)}{\text{Sum } (Pc.m. \quad Qa)}$$

$$\text{Type C} = \frac{\text{Sum } (Pi \quad Qc.m.)}{\text{Sum } (Po \quad Qc.m.)}$$

$$\text{Type D} = \frac{\text{Sum } (Pi \; Qc.m.)}{\text{Sum } (Pc.m. \quad Qc.m.)}$$

Type A is the Bureau of Labor Statistics formula or Fisher's formula Number 53 with broadened base, constructed by summing the products of prices times annual quantities marketed and dividing by the sum of the products of base prices times annual quantities. This index measures the fluctuations in value of a fixed "imaginary cargo consisting of specified quantities." [1] Its method of construction not only limits the number of commodities to be included to those that are marketed throughout the year, but it also fails to represent accurately the farmers' national wagonload which varies both in quantity and contents from month to month. The July wagonload, for in-

[1] Journal of American Statistical Association, September, 1923, p. 835, "Weekly Index Number and Farm Prices," Irving Fisher.

stance, contains a different mixture from that of the December wagonload.

Type B differs from Type A in that base prices for corresponding months are used instead of annual base prices. Comparison with base prices of corresponding months eliminates from this index whatever seasonal variation in prices obtained during the five-year base period. Unlike Index A, which requires a constant number of items throughout, Index B can include commodities marketed only during certain seasons, since the aggregate value for June, which may contain more or less items than the aggregate for May, is compared with a similar June aggregate at base year prices.

Type C is constructed by using annual base prices, as in Type A, but weighted by monthly sales instead of sales per annum. Each monthly aggregate consists of prices times quantities "normally" marketed that month and is divided by a similar aggregate for the base period, consisting of average annual base prices times quantities normally marketed that month. The resulting index is, therefore, a measure of fluctuations in the value not of an "imaginary" annual wagonload, but of a load "normal" for each month. On the theory that price level of a group of commodities on any given date is a function of quantities of commodities and total money units for which they were exchanged, and to the extent that monthly weights agree with quantities actually marketed in the given month, Index C is a measure of changes in price level, as distinguished from Type A which measures changes in value of a constant load.

Type D makes allowance for both seasonal variation in prices of the base period and the seasonal variation in quantities "normally" marketed. The aggregate value of the typical June wagonload, for instance, at June prices is compared with the aggregate value of the same June load at prices for the corresponding month in the base period. The resulting index thus represents changes in price level as measured against the "normal" for each month. In other words, seasonal price trend is here eliminated.

Index numbers constructed by each of these methods differ considerably. The extent to which they differ is shown in the following table, in reading which it should be remembered that differences between Type A and B, and between C and D are differences due to seasonal variation in base prices (allowed for in B and D. See formulae above). Differences between A and C, and between B and D are in each case due to the use of monthly weights (in C and D). The table is based on 156 comparisons of the monthly index numbers (1910–22). A and B differ by two points 38 times, and by three points

22 times; only once do A and B (also C and D) diverge as much as 8 points. On the other hand, comparisons between A and C, and between B and D show much wider divergence, the differences being due to the use of monthly weights. The comparison in the last column between A and D shows the distribution of differences due to both seasonal variation in base prices and seasonal variation in marketings.

TABLE I

DISTRIBUTION OF DIFFERENCES BETWEEN THE FOUR TYPES OF AGGREGATIVE INDEX NUMBERS*

No. of points of difference	Distribution of differences between indexes				
	A and B	C and D	A and C	B and D	A and D
(1)	(2)	(3)	(4)	(5)	(6)
1 and under	68	63	53	56	40
2	38	45	26	22	21
3	22	15	19	14	18
4	9	15	13	12	13
5	10	9	9	12	11
6	4	5	4	8	6
7	4	3	5	5	5
8	1	1	2	3	5
9	..	..	6	4	4
10	..	..	4	5	4
11–15	..	..	8	6	12
16–20	..	..	4	4	11
21–25	..	..	3	4	6
Total comparisons	156	156	156	156	156
Average number of points indexes differ from each other, 1910–22	2	2	4	4	6

* Farm prices of cotton, cotton seed, hay, flax.

The average of all monthly differences for the entire period due to seasonal variation in prices of the base period is approximately two points. (See columns 2 and 3 in Table I.) The average monthly difference due to monthly weighting is approximately 4 points (columns 4 and 5), while the average difference attributable to both seasonal base price and seasonal weights is 6 points (column 6).

Comparing the four index series for direction of price movement from month to month, it is found that monthly weighting produces more instances of differences in direction than does the elimination of seasonal price variation. Thus, out of 155 comparisons

A and B differ in direction 12 times, or 8% of total comparisons
C and D " " " 12 " " 8% " " "
B and D " " " 18 " " 12% " " "
A and C " " " 18 " " 12% " " "
A and D " " " 30 " " 20% " " "

Not only do the monthly index numbers thus differ in degree and in direction of price movement, but also the annual index numbers are subject to the same divergences. A study of the four formulae above will indicate that the yearly index for Type A and B is obtained by multiplying the *simple average* prices by annual quantities. The yearly index for Type D is obtained by summing the twelve monthly aggregates for the year and dividing by the similar sum for the base period. The effect of this method is that the yearly index is constructed on weighted average prices instead of simple average prices.[1]

The yearly Index C, on the other hand, can be constructed either by multiplying averages of prices by quantities per annum or by summing the twelve monthly aggregates. In one case it will agree with A and B; in the latter with D.

The yearly index numbers for a group of farm products [2] are presented below, together with a comparison of semi-yearly averages of

TABLE II

A COMPARISON BETWEEN INDEXES A AND D AND SEMI-YEARLY AVERAGES OF COTTON PRICES

Year	Index A Weight annual	Index D Weight monthly	Average price of cotton	
			January–June	July–December
			Cents	Cents
1910	111	112	14.2	14.6
1911	103	95	14.2	11.2
1912	90	92	9.9	11.4
1913	96	100	11.8	12.2
1914	88	78	12.1	9.1
1915	78	86	7.9	9.9
1916	113	126	11.5	15.5
1917	173	185	17.8	25.1
1918	231	235	29.4	29.6
1919	234	248	26.3	32.9
1920	244	213	36.8	27.4
1921	101	118	10.4	14.3
1922	145	152	16.4	21.2

prices of cotton, which is the predominating commodity in the group. It will be observed that the index numbers differ in direction (see 1915 and 1920), as well as in degree of price change. It will further be observed that Index D is higher than Index A whenever the prices of cotton during the last half of the year are higher than the prices during the first half and vice versa. This is due to the fact that cotton, as well as many other farm products, is marketed largely during the

[1] For 1920, for instance, the straight average price for cotton is 32 cents, weight per annum 5,688 million pounds, as used in Indexes A and B. In Index D, the sum of the monthly cotton aggregates is $1,565,000,000. This sum divided by the total quantity marketed for the 12 months, 5,688 million pounds, gives a weighted average price for cotton of 27½ cents.

[2] Cotton, cotton seed, hay and flax.

last half of the year. In a year of rising prices, therefore, Index D is higher than A and, in periods of declining prices, it is lower than A. Index D (and C) thus measures the annual price level more accurately than A, which, because of its per annum quantities, overweights prices during the first half of the year and underweights them during the last half.[1]

The differences so far shown between the four types of aggregative index numbers do not in themselves offer a basis for choosing the one best suited for measuring changes in farm prices. It is necessary to consider also the purpose of a farm price index. Its purpose may be to show (1) changes in value of a constant load of commodities, (2) changes in value of a constant load adjusted for seasonal price trend, (3) changes in the level of farm prices of commodities as they are "normally" marketed and (4) variations from the "normal" prices of commodities as they are "normally" marketed.

Obviously Type A fulfills purpose (1), but the load, the value of which it measures, cannot be made to include certain items of considerable importance to some sections of the agricultural population. Type B fulfills the same purpose, (2), modified, however, by the elimination of seasonal price variation. Requirements under (3) are met by Type C. This index, in so far as its monthly weights are approximately the actual quantities entering into trade during the given month, is a more accurate index of the level of farm prices than is Type A, which is admittedly "not strictly accurate [as a measure of price level] for any particular date." Type D fulfills the fourth purpose, that of measuring net changes in farm prices adjusted for seasonal price, and therefore indicates deviations from "normal" conditions of the corresponding period, or season.

[1] This holds true for prices of all commodities that are marketed in greater quantities during the fall months than during the first half of the year.

A NEW INDEX OF BUSINESS ACTIVITY

By Carl Snyder, *Federal Reserve Bank of New York*

The previous issue of this Journal contained a report upon the new Index of the Volume of Trade from 1919, which we have for some time had in preparation.[1] This new index represents a weighted composite of 56 different series available by months and grouped together in 28 major series, covering all the different fields of productive activity, primary distribution, distribution to comsumer, and financial and general business activity, which in their entirety make up the vast volume of the nation's total trade.

This new index sought to make extended use, for the first time, in index figures strictly comparable with the quantitative data in other lines, of the very valuable reports as to wholesale and retail trade which have been developed within the last few years by the Federal Reserve Banks and the Division of Research of the Federal Reserve Board, under the leadership of Dr. H. Parker Willis. And likewise this index attempted to utilize other available data expressed in dollar values, such as bank debits, building permits, exports, imports, new life insurance written, etc., the relative values or quantitative relations in which had been deeply disturbed by the extreme fluctuations in the various price levels within the last five years.

With the inclusion of all this new material, it was, in the minds of some, open to doubt as to just how accurately the new index reflected the actual variations in "trade" of the country, both in time and in percentages of variation. It was of interest, therefore, to discover other and independent measures.

Taking advantage of the new system of reporting debits to individual account inaugurated by the Federal Reserve Board, the Reports Department of this Bank has computed and for some time published [2] the velocity or rate of turnover of bank deposits in various cities of the country—a full report of which was given by Dr. Burgess in this Journal in the issue of last June.[3] These rates of turnover were computed according to a formula worked out by Professor Kemmerer, Professor Fisher, and Mr. J. H. Riddle, and the figures are available

[1] Carl Snyder, Journal of the American Statistical Association, December, 1923, p. 949.

[2] *Monthly Review of Credit and Business Conditions;* Federal Reserve Bank of New York; passim.

[3] W. Randolph Burgess, "Velocity of Bank Deposits," Journal of the American Statistical Association, June, 1923, p. 727; also, *Federal Reserve Bulletin* for May, 1923.

since the beginning of 1919. Having now completed five years, it was possible to estimate the normal seasonal variation for each of the cities, and thus make the index numbers strictly comparable by months.

As there was no discoverable secular trend, it was then possible to take simply the average velocity of the five years and compute the variations from this average.

The eight cities included, New York, Boston, Chicago, San Francisco, Buffalo, Rochester, Syracuse and Albany, gave an extremely

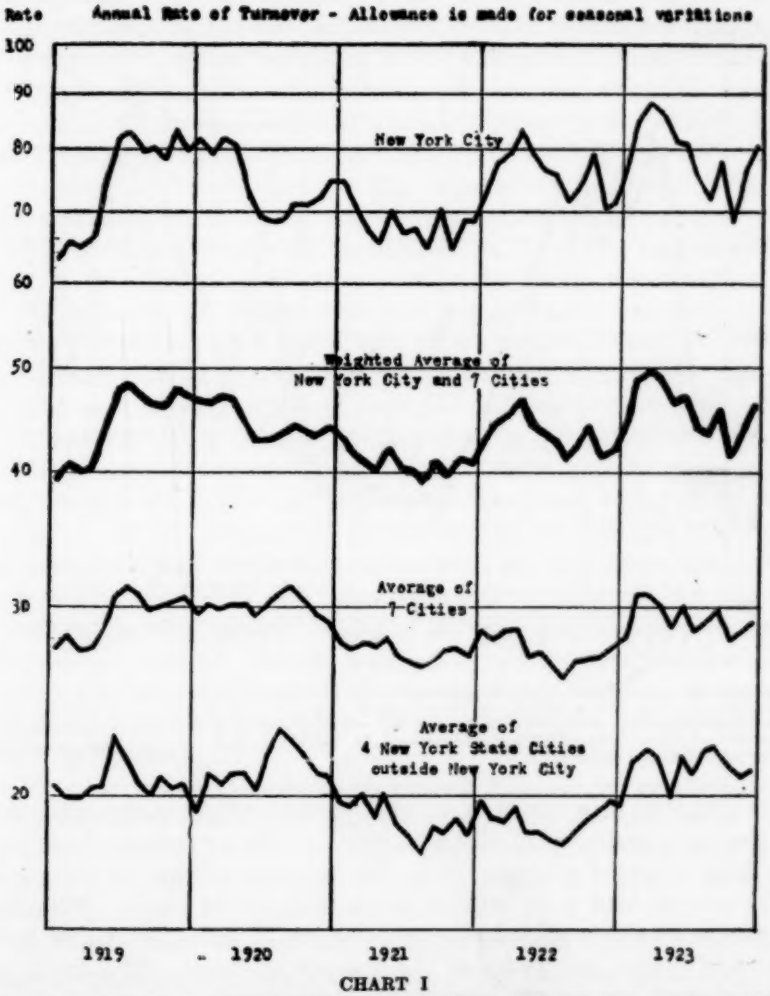


CHART I

Showing the variations in the rate of turnover in demand deposits in New York City and in a weighted average of New York and seven other selected cities, allowance being made for the usual seasonal changes. Comparisons of these variations with the five-year average are shown in Chart II.

good cross-section, it will be observed, both geographically and as to population, of the larger cities of the country. But as bank clearings, or, as we now say, debits to individual account, in New York City are on the average nearly equal to those of all the other clearing house centres of the country (and, roughly, to about one-third those of the whole country), an even weighting of the eight cities would be scarcely representative.

Accordingly the rates of turnover for New York City were given a weight of one against two for the average of the seven other cities,

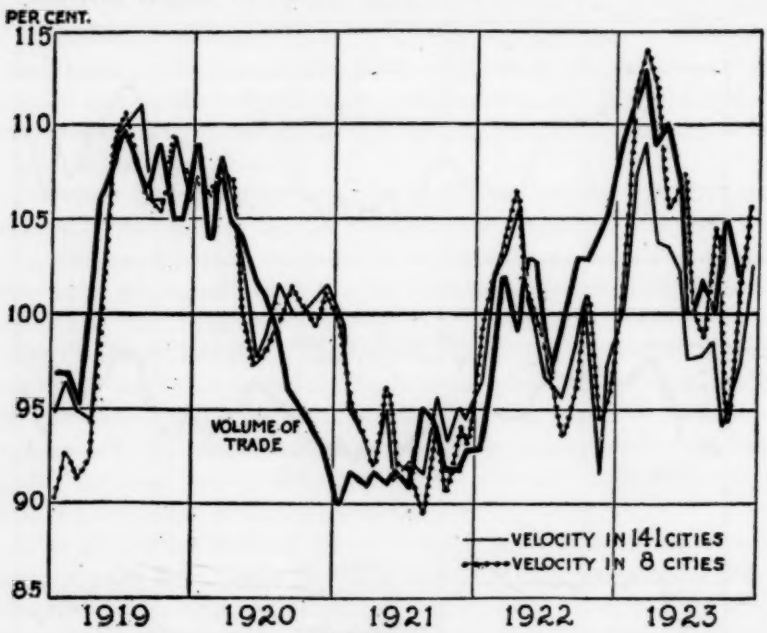


CHART II

Showing the rate of turnover of demand deposits in eight selected cities (dotted line), and in 141 cities (thin solid line) in comparison with the average for five years. With this is shown also the index of the Volume of Trade (heavy solid line), which is a weighted average of 56 independently computed series covering all the different fields of production, distribution, wholesale and retail trade, and general business and financial activity.

assuming that these cities fairly well represented the other two-thirds of the estimated total of all bank debits. The percentage deviations of these weighted averages from the five-year average of the whole gave a result that is set forth in the accompanying chart. With this is compared the weighted composite of the 56 series, which we have included in our new Index of the Volume of Trade. The result, as may be seen, was a quite remarkable concordance.

But it was still open to question as to whether the results for these

eight cities accurately reflected the variations in deposit activity for the country as a whole; and therefore whether the close correspondence revealed in this investigation might not have been accidental. It would be a heavy task to collate the deposits of all the banks in the 245 clearing centres now reporting. But it was possible to compare the deposits in the weekly Reporting Member Banks of the Federal Reserve System, with clearings in 141 leading centres; and this result, after eliminating the normal seasonal variation, is given in the dotted line in the same chart.

It will be seen that the variation in the rate of turnover for the 141 cities thus computed was, with a few exceptions, almost identically the same as for the weighted average of the eight cities for which an accurate computation could be made, which seems to leave no doubt as to the accuracy of the picture here presented. Doubtless the percentages of variation shown in the deposit turnover for these larger cities are somewhat, and it may be considerably, higher than is actually true for all the 28,000 commercial banks of the United States. But the time relations would probably still remain the same.

Now this, it will be observed, was precisely the conclusion reached as to the percentage of variation shown in our index of the total Volume of Trade. It was there explained [1] that almost all of the data available as to production of goods, distribution in wholesale and retail trade and other forms of business activity, are derived very largely from the larger industries, and in considerable part from the larger centres of the country; and that to this extent our Index of Trade is probably misrepresentative of the whole country. That is to say that if the index shows a swing from 10 per cent above to 10 per cent below the computed trend of past years, the reality for the whole country is without doubt somewhat less, and possibly very considerably less, than as here shown. Beyond question there is no such wide variation in the smaller industries and in the thousands of small towns and villages of the country, as is true of the larger, reporting industries and the larger centres.

We have reason, therefore, to believe that our composite for the total Volume of Trade, and the rate of activity in the turnover of bank deposits in the larger cities are quite closely comparable series, and the concordance found is therefore not so surprising as it might otherwise seem.

It is to be observed that this is the second confirmation of the findings of our Index of Trade and not the first. As was indicated in the previous article, it is possible to correct bank debits outside of New

[1] *l.c.*, p. 963.

York City for changes in the price level, by means of a weighted average of commodity prices, wages, the elements of the estimated cost of living and stock prices, and in this way obtain a line the variation from which, in the last five years, is practically identical with our Index of the Volume of Trade, alike as to time and percentages of deviation.

As here again, debits, or clearings, are reported only from the larger cities, this fact seems further confirmation of the view expressed[1] that if the variations in the larger industries and larger lines of business activity are certainly not more than 10 per cent above or below the computed normal, the reality, for the 110 million people of the country, a large part of whose activities cannot be here included, may be much nearer 5 per cent above and below the normal than is usually supposed.

We seem to have also definite confirmation of the result that the peak of business activity and of the total volume of trade, in the 1919–20 boom, came about July of 1919; and that this activity and volume was thereafter definitely declining, although for ten months longer prices continued to rise, and bank credit to expand, at an enormous rate. This unequivocal result, as portrayed in three separate indexes, seems, to the writer, of very high economic value. But it really only confirms what common sense would tell us, namely, that, when the whole labor capacity of the country is fully employed, at the maximum efficiency, no amount of mere additions to the currency or bank credit of the country, *i. e.*, to the nominal purchasing power, can increase the total of goods produced and exchanged, or, in fact, do anything but bring about a riotous rise of prices and wages and living costs, just as it did in 1920, with an inevitable collapse when the bubble of inflation is pricked.

But while common sense may tell us this, it was not in the least convincing to the vast body of manufacturers, distributors, and other business men caught, in that period, in the delusion that rising prices and big paper profits meant a real expansion of trade; and it might be no more convincing if we should again have some kind of repetition of this same disastrous experience. It seems, therefore, of real value that we now possess what seems to be complete and indubitable proof; and, furthermore, that we now have means of determining when this point of maximum production and trade, for a given period, has been reached. We no longer have the excuse of ignorance or uncertainty.

It seems equally clear, from comparison of the three indexes under view with various indexes of production, that indexes of production alone, especially as they are largely derived from the great basic in-

[1] Carl Snyder, "Shall We Modify Our View of the Business Cycle?" *Administration*, May, 1923.

dustries, are not, as has been assumed, a reliable index of the volume of trade or of "business activity," either in their time relations or in the percentages of variation shown. And the same appears equally true of composite indexes derived from railway traffic, factory employment and the like.

It may be added that, with the accurate determination of the variations in the volume of the country's trade, it now becomes possible, from the bank debits, to compute the general price level, as contrasted with commodity prices at wholesale and other price indexes; and likewise to estimate the reliability of even our best commodity price indexes in times of great dispersion, as in the last eight years. These matters will be dealt with in a future article.

THE NATURE OF AN UNEQUIVOCAL PRICE-INDEX AND QUANTITY-INDEX

By Sir George H. Knibbs

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22. Application of price-indexes to questions of cost-of-living.
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24. The equivocal character of all other price-indexes.
25. Confused indexes.
26. Independent agreements of price-indexes.
27. Comment on so-called ideal-indexes.
28. Conclusion.

1. Introductory. A large literature[1] exists as to the nature of price-

[1] Already there are excellent bibliographies; for example that given by Mr. C. M. Walsh in *The Measurement of General Exchange Value* (Macmillan, 1901), Professor Irving Fisher in his various books, particularly "The Making of Index Numbers" (Houghton Mifflin Co., 1922).

The general principles of computing price-indexes have been set out by me in Appendix IX of Labour

indexes and quantity-indexes, the subject having often been elaborated, however, with but little regard to its essential simplicity, or to the need of clear and unequivocal conceptions as to the real nature of any type of index appropriate to a particular application. This article aims to show that a properly ascertained index may be readily understood by the "man in the street," and may be very easily computed, and that it is preferable to adopt a technique for computing it, which has at least this as well as other merits. It seems almost superfluous to say that, provided they are correct, the use of formulae, the exact significance of which is definite, is preferable to the use of formulae, the exact significance of which is obscure.

2. Unequivocal Indexes. In order to obtain an exact and quite unequivocal conception of the whole matter, let us suppose, since an index must relate to some reality, that the usage of a particular community embraces commodities A, B, C, etc.; and that, when the quantities used in some given unit of time are Q_a, Q_b, Q_c, etc., their prices per unit of quantity are P_a, P_b, P_c, etc. Let us suppose further that during some other period (an equal unit of time), the quantities and prices of the commodities are respectively Q', P', etc., with suffixes as before to denote the particular commodities in question. Then $\Sigma(P\ Q)$ and $\Sigma(P'\ Q')$ are respectively the aggregate-values of the series of such commodities used on the two occasions. These two sums or aggregate-values, however, in such form, are significant only as totals, and have no other particular significance, since the ratio of one to another is affected both by price and by quantity.

Suppose however that, for each commodity, $Q'=Q$, then the fraction $\Sigma(P'\ Q)/\Sigma(P\ Q)$, viz., the ratio of aggregate-value for the second unit-period to the aggregate-value for the first unit-period is no longer merely a ratio of totals, it also shows unequivocally the effect of the change of price. Thus it is an unequivocal price-index for the quantitatively-unchanged complex of commodities, A, B, C, etc.

It is obvious that if the quantities were different on the two occasions, and if at the same time the prices had been unchanged, the preceding formula would become $\Sigma(P\ Q')/\Sigma(P\ Q)$. It would be still the ratio of the aggregate-value for the second unit-period to the aggregate-value for the first unit-period. But it would be also more than this. It would show in a generalized way the ratio of the quantities on the two occasions. Thus it is an unequivocal quantity-index for

and Industrial Report, No. 1, Dec., 1912, Australia, "On the establishment of a basis for international comparisons of the exchange values of gold, and variations in the cost of living," pp. xxxix–lxii; and in Report No. 9, July, 1919, Part I, Introductory remarks on price-index, pp. 176–180, Part II, The purchasing power of money and the nature of price-indexes, pp. 181–198, Part III, Technique of computing price-indexes, pp. 199–222, Part IV, The significance of price-indexes and conclusions, pp. 223–229. In Appendices II and III, the indexes furnished by a large number of countries are also given.

the complex of commodities, unchanged as to price and differing only as to quantity.

Let it be noted that the mere algebraic form of these expressions shows at once that the logic of the problem of finding these two indexes is identical. In the former case, in which the quantities remain unchanged, we discover the ratio of what may be called the generalized-values or price-aggregates. In the latter case, in which it is the prices that remain unchanged, we discover similarly the ratio of the generalized-quantities or quantity-aggregates. These propositions are the essence of the whole matter. They reveal the nature of an index, and show that its proper significance is immediate and unequivocal. It is, in short, the ratio which the total values of a series of commodities have when either the prices alone, or the quantities alone are changed.

3. Reversibility of Properly Deduced Indexes. These algebraic forms, for the computation of an index, reveal also the fact that it is immaterial whether we take the first or the second date as the basic date (*i. e.*, date of reference), for if I denote the deduced index, we have necessarily, as clearly we ought logically to have

$$(1) \qquad \ldots \quad I_{01}=1/I_{10},$$

and this is immediately obvious from the preceding expressions. In other words, the characteristic of reversibility applies to indexes when they are calculated in the manner indicated. It needs hardly be said that every properly deduced index must possess this characteristic.

4. Units Used Immaterial. It remains to note that the price P is for a unit of quantity, and reciprocally the quantity Q is that of which the price of a unit is P; that is to say, $P_k\,Q_k$ is the value of the amount K say, used of any given commodity, in whatever unit of time is adopted. Self-evidently the units in which P and Q are expressed are quite immaterial provided only they are the same at the different dates for which the comparison is to be made: thus they may be pence, shillings, cents or dollars, etc., and pounds, hundredweights, grams or kilograms, etc.; litres or quarts, etc., or may be single articles, dozens or scores of articles, etc. In short $\Sigma(P'\,Q')/\Sigma(P\,Q)$ is the same quantity or ratio as $\Sigma(xP'\,yQ')/\Sigma(xP\,yQ)$, where x and y are any common multiples whatsoever.

The question of the lengths of time over which quantities like P and Q are practically estimated will be considered later. It will suffice to say here that the adopted unit may be what we please, a minute, a day, a week, or a year.

5. Equivocal and Unequivocal Results. What has been stated shows

quite definitely that, when the rate at which the quantities of a series of commodities used by any community is constant, the price-index based thereon is definitive, or, in other words, it is quite unequivocal, assuming, of course, that the prices themselves are correct. Also if the prices do not change, or are identical for two compared periods, then the quantity-index is definitive, that is the quantity-index unequivocally measures the quantitative ratio of the complex quantity-aggregates.

It will be convenient to call the series of prices and quantities for a given number of commodities, characterizing the usage of a given community, respectively the price-regimen and the quantity-regimen for that community; that is,

P_a, P_b, P_c, etc., is the price-regimen for commodities, A, B, C, etc.

Q_a, Q_b, Q_c, etc. is the quantity-regimen for the same commodities. Since any aliquot part of these gives the same results relatively, we shall use the small letters p and q in the same sense as the capital letters, and for the sake of precision shall regard p and q as the price obtaining, and the quantity used, during any small but definite period of time, say a moment. To be definitive, therefore, a price-index presupposes two things, viz., (i) that the quantity-regimen is identical for the moments at which the prices are ascertained, and (ii) that it correctly represents the usage of the community. Similarly to be definitive a quantity-index presupposes (i) that the commodity-prices obtaining at the two compared moments are identical, and (ii) that they really represent the facts for any community in question. In such cases the results are definitive or unequivocal; in all other cases they are equivocal or not definitive.

6. In What Sense Certain Results Are Equivocal. Professor Irving Fisher has very aptly called the quantitative complex of commodities, or regimen, a "composite unit." In order to be quite definite, we shall call it a quantity-regimen; and similarly a series of fixed prices we shall call a price-regimen (see Section 5). The distinction between definitive and equivocal results can then be thus envisaged:

When for different dates the actual prices of a series of commodities are attributed to the constant usage-quantities or quantity-regimen common to the different dates, the ratios between them constitute a definitive series of price-indexes, and the reciprocals of these denote definitively, and for the dates in question, the relative purchasing-efficiency of money, measured by the particular quantity-regimen adopted. Since, however, if both prices and quantities are varied, there will then be a manifold infinity of ways in which $\Sigma(p\ q)$ may be

made equal, the price-indexes lose their precise meaning, and the exact basis on which the relative purchasing-efficiency of money is ascertained ceases to be sharply defined.

This may be stated in another way. The measurement of the relative purchasing-efficiency of money at different dates is necessarily dependent upon the adoption of a particular quantity-regimen as a regulative basis, and is exact only in relation to that particular regimen. A similar proposition obtains in regard to measurements of the ratios between quantity-aggregates. If the price-regimens at different dates are identical, then the ratios, subsisting between the aggregate-values found by attributing these constant prices to actual quantities, express their general quantitative relation.

7. Simple Illustration of the Condition Giving an Unequivocal Character to Indexes. Very little consideration is necessary to show that indexes are exactly significant only when their regulative bases, or the regimens by means of which they are calculated, are invariable. If from time to time an individual purchases the same quantities of a definite series of commodities, the reciprocals of their aggregate cost to him disclose the changes in the purchasing-efficiency of the money he is using to buy them. Thus if he pays at one date twice as much as he paid at an earlier date, the purchasing-power of money has in the interval fallen to one-half. But, if the definite series of commodities and quantities (the regimen) is not adhered to, then the aggregate cost is affected not solely by the change in purchasing-efficiency of money, but also by the change of the regimen itself. (An analogous proposition holds in regard to quantitative comparisons.) The price-index on an indefinite or variable basis cannot possibly yield an unequivocal result, for it reflects two things, viz., change of price and change of quantity, whereas the whole purpose of a price-index is to reflect the effect of change of price solely. Indeed the very conception of a price-index is that it shall be free from the influence of change of the quantities in the regimen; and similarly of a quantity-index that it shall be free from the influence of change in prices.

In many other approaches to this subject, and in other modes, actually used, of ascertaining indexes, the technique or algorithm of computation disguises the fact that this confusion exists, viz., that arising through the effect of the double change (viz., in both quantity and price). Such confusion exists, but is lost sight of. Obviously, then, the only formulae which give absolutely unequivocal results are the following or their algebraic equivalents, viz.:

(2) . . . $I_{01}=\Sigma(p_1q)/\Sigma(p_0q)$; (2a) . . . $Q_{01}=\Sigma(pq_1)/\Sigma(pq_0)$,

I denoting the price-index computed from prices at dates 0 and 1,

with the first as the basic date and referred to a constant quantity-regimen; and Q denoting the quantity-index computed from the quantities at dates 0 and 1 with the first as the basic date, and referred to a constant price-regimen. It remains to consider the proper quantity-regimen to be adopted to compute I, and the proper price-regimen to compute Q.

8. The Optimum Quantity-Regimen. It might at first sight be thought that we ought to include all commodities used by a community, but for several reasons this is not practicable, viz.:

(a) The enormous number and variety of the commodities;
(b) The uncertainty as to their identity in quality, price, etc.;
(c) The negligible influence of some of them upon the result.

These are the main reasons for limiting the number of commodities selected for a regimen. It may be added that an examination of the uses to which price and quantity indexes are to be put will disclose the fact that we ought to consider the purpose to be served in their application to economic questions. The purpose of an index is governed, of course, by the range and character of its application. For example, it may be required to apply to the commodity-usages of a whole community, or, on the other hand, merely to a section thereof; in extreme cases to the usage of a very small group of persons, a family, or even an individual. For general purposes the convenient conception is the usage for some unit of time, which unit is quite arbitrary, of the "average man" of a particular class, or of the entire community, as the case may be.

It is easy to see that even in the one community, appropriate price- and quantity-indexes are specifically different for the rich, the middle-class, or the poor, for people of the town or country; for those who live on special diets, or who make particular commodities (*e. g.*, clothing) of special importance in their schemes of personal expenditure. So too for nations, the regimen of an omnivorous nation, being totally different to that of a nation whose diet consists largely of fish and rice, the price- and quantity-indexes appropriate for the one would be quite inappropriate for the other.

Although it is true that special regimens are needed for special classes of peoples and for special purposes, there are two which on account of their broad sphere of application demand more than ordinary attention. These are such as are concerned with retail operations in commodities, and such as are related to wholesale operations. From the practical point of view the optimum regimen in both cases is that which is restricted to the series of commodities most

largely used, and the character, qualities and prices of which are accurately ascertainable. Some commodities may have a relatively large exchange-value, and yet for such index-purposes as we are considering be of no moment, for example, gold (or the basic metal of the metallic currency), precious stones, jewelry, fine wines, "delicatessen," luxurious textiles, *objets-d'art*, books, etc. The reasons are practically self-evident. The basic metallic-currency does not change its price, and in relation to indexes is in the general sense not a commodity; precious stones and jewelry play an insignificant economic rôle in community-usage generally, as also do wines, "delicatessen," *objets-d'art*, etc.

When price-indexes are used in order to measure fluctuations in the purchasing-efficiency of money, the prices should of course refer to identical commodities, that is, A on the first occasion must be identical with A' on the second. In short the series of commodities must be the same at the compared dates, otherwise the results would be vitiated by confusions of kind, quality, etc. One illustration will suffice to make this quite clear. Suppose sugar to be the commodity in question, the grade affecting its price; then its grade or quality must be identical for both dates, otherwise the aggregate-values from which the index is computed are prejudiced. In fact, a different commodity is really being taken into account. More generally, in order that indexes shall accurately reflect the varying purchasing-efficiency of money, the quantity-regimens used for comparisons must be identical as regards intrinsic character and quality. The optimum quantity-regimen is therefore that which is:

(a) sufficiently comprehensive,
(b) readily identifiable as to intrinsic character, and
(c) omits all commodities of negligible moment.

If these conditions are not properly attended to, the deduced indexes do not adequately and accurately represent the commodity-usage of the community, do not correctly represent change in the purchasing-efficiency of money, and become unnecessarily tedious to calculate. In actual practice many indexes are of inferior value, because of inattention to these desiderata. This fact can very easily be masked by the adoption of bad methods of computing.

9. Real and Pseudo-Identity of Quantity-Regimens. If at two dates the commodity usage of a community has not changed, one has for the purpose of practical computations simply to be assured of the identity of the commodities (of the elements of the regimen) for which the prices are ascertained. This practically is by no means uniformly

easy, especially as regards the quality of the commodity (*e. g.*, tea or textiles used in two different localities). When in respect of its intrinsic nature and its quality a commodity is the same at two dates we may call it a case of real identity. When, however, it is undifferentiated merely in respect of name, but is sensibly different intrinsically, or in quality, the commodity on the second date replacing that of the first date in the community-usage, it is a case only of pseudo-identity. For example, in textiles a tweed at the second date may have the same general appearance and weight and replace the use of that at the first date, but the number of threads in the warp and woof may be wholly different and this may, of course, have affected the price. Such a case of pseudo-identity may be treated in one of two ways according to the purpose of the investigation. For one economic purpose it may be regarded as not a case of identity of regimen, since the difference in price is due to difference in quality, and would vitiate the deduction of a change in the purchasing-efficiency of money. For another economic purpose, however, we may ignore this difference, and treat it as if it were identical, and deduce the purchasing-efficiency of money regarded as under the compulsions of social habit, which, be it said, are continually replacing the usage-commodities of any date by others of different quality, or even of different character.

We see from these and similar considerations that mere mechanical directions as to the computations of price- and also quantity-indexes cannot be adequate guides. The purpose of every inquiry has to be intelligently conceived, and its effect upon the adopted regimens has to be appropriately studied; the mere technique of computation is only ancillary to the end in view.

10. Real and Pseudo-Identity of Price-Regimens. The relations between the prices and the intrinsic characters—including quality—of a series of commodities are such, that for the unequivocal deduction of quantity-indexes, there must in strictness be real identity. The remarks of the preceding section apply, *mutatis mutandis*, to this case. In the former case we are concerned with the problem of a general price-measure, viz., the aggregate value of a fixed "composite-unit"; in the latter case the problem is to find an appropriate quantity-measure between two composite aggregates, the prices of the elements of which are treated for the purpose of such a measure as invariable. The significance of the latter case is, however, not so readily represented as the former, but it may be easily seen that the mere ratio of the aggregate-values is not a measure of their relative quantities since, for example, that difference may be wholly due to change of price. Keeping the prices constant has the effect of eliminating this cause of

difference, and furnishes the best possible measure of the quantity-change in this group of commodities.

11. Modes of Ascertaining Prices and the Measure of Their Precision. Prices may be those as they exist at a given date, or may be averaged over a period. The sources of information are so chosen by statisticians that in so far as possible each of the prices furnished for a given commodity presumably relates to a numerically equal section of the community in which case, if n values of a price had been obtained, the required average would be:

$$(3) \qquad \ldots \quad p=(p'+p''+p'''+\text{etc.})/n.$$

The differences from the mean price show the "scatter" of the prices about that mean, and the average of these differences, m, taken without regard to sign, that is,

$$(4) \qquad \ldots \quad m=\pm\{(p'-p)+(p''-p)+(p'''-p)+\text{etc.}\}/n$$

may be called the "mean scatter" or the scedasticity of the price p.

If the populations of the sectional divisions (s) of the community represented by these prices were not equal, the more tedious formula,

$$(5) \qquad \ldots \quad p=(s'p'+s''p''+s'''p'''+\text{etc.})/(s'+s''+s'''+\text{etc.})$$

would be required, but in all practical cases only the former, (3), is needed. If, however, we put $S=ns=s'+s''+s'''+\text{etc.}$, that is if S is the total population of the community, then the scedasticity, σ, of the sectional divisions of the populations is similarly

$$(6) \qquad \ldots \quad \sigma=\pm\{(s-s')+(s-s'')+\text{etc.}\}$$

Owing to the fact that in practice the equivalence of the sectional divisions is a matter of conjecture, even if the prices are exactly given, the probable error increases with σ. Again if the sectional divisions are exactly equal, any error in the price furnished as the mean or the mode for the sectional division makes the error probably increase with m. In general the error may be said to increase with increase of both quantities. The value of σ is not generally ascertainable, however; m is, of course, always ascertainable. It may readily be shown that the errors consequent upon these limitations are very small. Thus if $v^N=p-p^N$, and we make the extreme assumption that the deviations from the mean price are analogous to deviations in observations of a physical quantity from the mean value, in which case we might write the "probable error," ρ, as

$$(7) \qquad \ldots \quad \rho=0.845m/\sqrt{n}; \text{ or better}$$

(7a) . . . $0.845\Sigma v/(n\sqrt{n-1})$ Peter's formula, or

(7b) . . . $\rho=0.674\sqrt{\{\Sigma v^2/[n(n-1)]\}}$, Gauss' formula.

Thus, even the values of ρ are small. The real probable error is actually very much smaller however, $\kappa\rho$ say, where κ is a small fraction. The inherent imprecision of the data makes it quite indifferent which of these formulae (7), (7a) or (7b),[1] is used and in any case we may easily satisfy ourselves that the effect on the index computed is negligible.

The preceding remarks relate to prices at a given date. Suppose, however, we desire the average price over a period, which is virtually involved when prices are obtained in reply to the question: "At what price have you sold the largest quantity of commodities *A*, *B*, *C*, etc., during the last week (or month, etc.)?" or "What price have you obtained, on the average, for commodities *A*, *B*, *C*, etc., during the last week, etc.?" The former question, based on the mode of the price-distribution as it has been observed by the informant, is ordinarily the more reliable; the latter is apt to be answered by a mere guess at the average. It is on the assumption that each informant gives a price which may properly be regarded as applicable to an equal section of the community that the mean of the prices so obtained is taken as the basis of computations; and, as shown, m is to be regarded as furnishing merely some sort of indication of the possible range of error, not as actually the probable error.

12. The Precision of the Aggregate Value of Any Regimen. If the average deviation m_a, m_b, etc., of the prices of each commodity be regarded as actually the measure of its error—again an extreme assumption—the corresponding total error probable M would be

(8) . . . $M=\pm\sqrt{\{q^2_a m^2_a+q^2_b m^2_b+\text{etc.}\}}$; *i. e.*$=\pm\sqrt{\Sigma(q^2m^2)}$.

Thus the uncertainty (μ) of a price-index due to this is:

$$(9) \quad . \ . \ . \quad I_{01}\pm\mu_{01}=\frac{\Sigma(p_1q)\pm M_1}{\Sigma(p_0q)\pm M_0}=\frac{\Sigma(p_1q)}{\Sigma(p_0q)}\cdot\frac{1\pm M_1/\Sigma(p_1q)}{1\pm M_0/\Sigma(p_0q)};$$

After some substitutions, this expression gives for μ

(10) . . . $\mu_{01}=\pm\{\sqrt{(M^2_1+I^2_{01}M^2_0)}\}/\Sigma(p_0q)$;

and this would be the uncertainty for the price-index as computed on the extreme assumption indicated. Similarly for the reciprocal index $I_{10}=1/I_{01}$, we have

[1] To illustrate by an extreme example:
Prices say 13, 14, 16, 16, 17, 20; Sum=96; Average =16=*p*.
Differences 3 2 0 0 1 4; Sum=10; Average 1.67 =*m*; $0.845m=1.41$.
Probable error $=1.41+\sqrt{6}=\pm0.57$; by Peters' formula $0.845\times10+6\sqrt{6}=\pm0.63$.
By Gauss' formula $0.674\sqrt{\{(9+4+0+0+1+16)/(6\times5)\}}=\pm0.67$.

$$(11) \quad . \; . \; . \quad I_{10} \pm \mu_{10} = \frac{\Sigma(p_0 q) \pm M_0}{\Sigma(p_1 q) \pm M_1} = \frac{\Sigma(p_0 q)}{\Sigma(p_1 q)} \pm \frac{\sqrt{(M^2_0 + I^2_{10} M^2_1)}}{\Sigma(p_1 q)}.$$

If μ be multiplied by 0.845, one obtains with sufficient precision the "probable error" of the index but only on the extreme assumption. Because, however, this is much greater than the actual probable error the quantities M and μ are of theoretical rather than of practical interest. Moreover the data are often hardly precise enough to justify attaching very high importance to minute differences. They nevertheless are of some service in enabling a definite idea to be reached as to the accuracy attained. By computing them, one is enabled to get a rough idea as to the degree of precision to which results may be appropriately expressed, for example, that it is useless to attempt a higher precision than say 1 in 100, or 1 in a 1,000, etc.

13. The Effect of Price-Uncertainties on the Quantity-Index. The ascertaining of the precision of the aggregate-value of a regimen is also necessary if it be desired to determine the accuracy of the generalized quantity-measure or quantity-index for two given dates. In this case the price-regimen is fixed, the uncertainties are assumed to be the same for the two series of quantities. Hence, putting N for the quantity analogous to M in (8), we have

$$(12) \quad . \; . \; . \quad N_0 = \pm \sqrt{\Sigma(q^2_0 m^2)} \text{ and } (12a) \quad . \; . \; . \quad N_1 = \pm \sqrt{\Sigma(q^2_1 m^2)}.$$

N_0 is the same as M_0 in (8) and (9) etc. Calling the uncertainty of the quantity-index ν, we have similarly to the preceding case, viz. (9),

$$(13) \quad . \; . \; . \quad Q_{01} \pm \nu_{01} = \frac{\Sigma(pq_1) \pm N_1}{\Sigma(pq_0) \pm N_0} = \frac{\Sigma(pq_1)}{\Sigma(pq_0)} \pm \frac{\sqrt{(N^2_1 + Q^2_{01} N^2_0)}}{\Sigma(pq_0)},$$

the expression for ν being, in form, but not in value, identical with (10), as obviously it should be. These results are very readily calculated; and, as before said, though they are not required for the computation of indexes, they are requisite for any critical review of their value when ascertained.

If the real "probable errors" were known for each price (*e. g.*, $p' \pm \epsilon'$, $p'' \pm \epsilon''$), the probable accuracy of the whole would be determinable. Thus let the aggregate value and its probable error be denoted by $T \pm t$, and let i/T be denoted by τ, then, the value of the index is

$$(14) \quad . \; . \; . \quad I_{01} = (T_1 \pm t_1)/(T_0 \pm t_0) = \left\{1 \pm \sqrt{(\tau^2_1 + \tau^2_0)}\right\} T_1/T_0.$$

14. The Determination of the Quantities in a Regimen. When the quantity-usage of a people is required we may have perforce to treat the matter in a large way, for example, in ascertaining the consumption

of each commodity by adding production to imports and deducting exports, taking carefully into account all initial and terminal uncertainties. This may be called the gross-usage method, and from it we can obtain the average usage of an "average" individual for any given unit of time. Another method is the budget-method, viz., the usage of commodities by a suitably chosen and sufficiently large group of individuals. If the number of these is known, and also the length of the period over which observation has extended, the average usage per individual per unit of time can also be found. A practical statistician will of course use both methods, as far as practicable.

Probably for all larger purposes it is desirable to adopt the commodity-regimen deduced over a considerable period of time, say five years, for investigations into general changes in the purchasing-efficiency of money. Otherwise the results become meticulous and one is liable to introduce minor fluctuations which it were preferable to ignore. For example, the food-requirements and food-usage of any community has periodic (annual) variations, as well as irregular ones, the former due to seasonal changes and the latter to variations in the quantities of commodities available at suitable prices. It is easy to see that these things do not affect uniformly the various classes in the community, hence they produce a differentiating effect on usage as between one class and another. Much of this we may regard as only of minor or of family or personal moment, to be met by adjustment as between regimen and available income; but even so there are outstanding variations which exist for the community as a whole. For this reason considerations of the use to which indexes are to be applied govern the propriety of the adoption of particular regimens.[1]

15. Variations of the Regimen Itself and Their Effect. The greatest practical difficulty in deciding upon a regimen for measuring the purchasing-efficiency of money is that of distinguishing between a real change in purchasing-efficiency and change due to variation in the general standard of living. This may now be considered. The changes to be taken into account for any one community are as follows:

(i) Usage is subject to periodic, temporary and permanent changes.
(ii) These changes may be variations of quality, of degrees of usage, or substitutions.
(iii) Or they may be introductions of new commodities.
(iv) Communal well-being or poverty and public opinion may reflect itself in variations of the regimen.

As between one people and another it may be said that the national

[1] Unreasonable claims are sometimes made for the adoption of a particular regimen, urging that it should govern economic deductions, without regard to world-conditions fundamentally affecting them.

dietary and other habits so differ that a regimen appropriate to the one may be wholly inappropriate to another for almost all purposes for which the indexes may be domestically required. It is desirable before considering these wider matters to examine more closely the question of the adoption of any particular regimen for a given community. We have seen that its elements, even in the best of circumstances, have some margin of uncertainty (see formulae (5) to (13)). We may graphically represent this for a line, showing the fluctuations of a price-index or a quantity-index, by a penumbra, the width of which is the real "probable error" on either side of the line denoting the changing index. Variations which lie within the penumbral region may be regarded as relatively of small significance.

16. Continuity of a Series of Price-Indexes. From what has been indicated it is evident that a series of price-indexes are strictly continuous only if the quantity-regimen used to compute them remains unchanged. When such a regimen is appropriate, that is to say for so long as it represents with sensible precision the commodity-usage of a given community, it is possible to have practically this strict continuity of price-indexes. But if usage changes the indexes cease to be appropriate, since they have reference to a commodity-usage which has become fictitious. Let us suppose that at series of given dates 0 to n the rates at which commodities were being used were q_0 to q_n. To ascertain the best average-quantity over all we must make some assumption as to the law of change of such usage. Thus suppose the quantities of each commodity are $q_0, q_1, q_2 \ . \ . \ . \ q_n$, these suffixes denoting dates. If we are justified in supposing that the curve of change in the value of q is representable by $\alpha+\beta t+\gamma t^2+$ etc., we may very readily find the average value of q for any period. Thus it may be found from the following formulae:

(15)
Range $n=0$ to 1. Range $n=0$ to 2. Range $n=0$ to 3.
Mean $=\frac{1}{2}(q_0+q_1)$; $\frac{1}{6}(q_0+4q_1+q_2)$; $\frac{1}{8}(q_0+3q_1+3q_2+q_3)$;
Range $n=0$ to 4.
Mean $=\frac{1}{90}(7q_0+32q_1+12q_2+32q_3+7q_4)$.
Range $n=0$ to 5.
Mean $=\frac{1}{288}(19q_0+75q_1+50q_2+50q_3+75q_4+19q_5)$; and so on.

These formulae are, however, tedious and the assumption that the variation in the value of q is of the continuous nature supposed is an unnecessary refinement. Hence for practical purpose we may probably always use, for any range 0 to n

(16) $\ . \ . \ . \ q=(q_0+2q_1+2q_2+ \ . \ . \ . \ +2q_{n-1}+q_n)/2n.$

and regard this for each element as the appropriate quantity to be

used for indexes covering the whole period. This gives a strict continuity to the series of price-indexes, the actual significance of which, however, will be limited in the measure of the deviation of q from the mean so found. If the deviations are irregular it is probably satisfactory; if they progress systematically it is not.

If we proceed in the way shown hereunder, however, the strict continuity is lost and we obtain only a pseudo-continuity.

Quantity of each commodity

$$q_0;\ q_1;\ q_2;\ q_3;\ \text{etc.}$$

Quantity adopted in computation of price-index

$$\tfrac{1}{2}(q_0+q_1);\ \tfrac{1}{2}(q_1+q_2);\ \tfrac{1}{2}(q_2+q_3)\ \text{etc.}$$

In other words, by adopting the arithmetic mean of the quantities for dates k and $k+1$ to find the $(k+1)$th index, we at least get some kind of continuity. If the changes in q may be assumed to be linear in each case, the best basis for computing that index, and the successive indexes are well found, but the $(k+1)$th index is *not* referable to index 0, but to the kth index. This is obvious when we consider an extreme case where the regimen has become very different at $k+1$ to what it was at 0. We shall now consider whether there be any remedy for this. The matter is important, yet much popular confusion exists in regard thereto. For this reason a restatement of the conditions which ensure an unequivocal price-index is not superfluous.

17. Analysis of the Consequences of Change of Regimen and Change of Price. We have seen that the essential of a properly ascertained price-index is that it shall not be vitiated by mere change of regimen. In short only if a regimen is unchanging in respect of the character and quantities of its elements, can it furnish—through its aggregate-value or price—an unequivocal measure of changes in the value of the purchasing-efficiency of money. It is in short the "composite unit" by means of which the change is measured. And though each individual, each class in a community, and each community as a whole, has its characteristic regimen, and in some cases these may differ widely, it gives a result which may be taken as sensibly true for the community as a whole. It is only in so far as the composite-unit represents actual average usage, that it is unequivocal. If the composite-unit is not an unchanging one, the index becomes misleading, simply because the values from which it is deduced are affected by the change. It is self-evident, therefore, that a quantity-regimen or composite-unit for estimating changes in the purchasing-efficiency of money depends upon the following, viz.:

(i) That in respect of its field of application the elements are sufficiently comprehensive, *i. e.*, that they sufficiently cover the range of commodities actually used by the particular community to which they refer.

(ii) That, relative to each other, the quantities of the specified elements of the regimen represent the actual usage of them by this community.

(iii) That the prices, per unit of quantity, correctly represent—on the average—the prices paid by the community.

(iv) That for any two dates compared, and in order to guard against the vitiation of the result by "change of standard" the elements (commodities) are properly identified (*i. e.*, with sufficient accuracy).

(v) That the regimen (*i. e.*, its elements) remain the same at the dates compared, that is either actually, or with sufficient approximation for practical purposes.

It is self-evident that condition (v) may be inconsistent with condition (ii), for the reason that any community may—and in fact does—change its usage, owing to changes of season, to variations in price, to mere changes of habit, etc.

Again it is also self-evident that the best basis of comparison is a regimen which differs the least possible amount from the actual regimens on any two dates compared. For each individual element this is, of course, the mean of the usage on the two occasions, that is $\frac{1}{2}(q_0+q_1)$; and assuming a linear change in the quantities there can be no better basis of comparison. But the question we have asked is this, viz.: "Whether, although this is the best possible basis for each element, the whole group of these means is also the best possible group for the comparison of the aggregate values which furnishes the price-index." The answer is "yes," only if the "scatter" of the quantities about an average is irregular and small.

Before dealing further with the question of unavoidable changes of regimen it is important to consider the price-ratio method of computing indexes.

18. The Price-Ratio Method of Deducing a Price-Index. Economists apparently have always had some degree of conviction that deductions of price-indexes ought to take account of all the evidence, and that, whenever a ratio between the price at two dates, k and $k+1$, can be obtained, the ratio p_{k+1}/p_k ought to be included for what it is worth in the scheme for ascertaining the index.

The rationale of the price-ratio method may be thus stated: Every price-ratio affords an indication—*pro tanto*—of the change taking

place in the relation between money and commodities. But such ratios are not all of equal importance. If, then, each be weighted in proportion to its economic importance as measured by its money-value at some date taken as a basis for comparison, then the ratios of the values of each to the value of the whole give fractions (weights) the sum of which is unity, so that the price-index is a weighted average of the price-ratios, the weight being that at the basic date. This statement is not sufficiently adequate to be rigorously accurate; the regimen applied to measure the economic weight must be sensibly correct.

First of all it is important to observe that the question of identity in character and quality of the commodities of which the price-ratio is found, is no less imperative than in the aggregate-regimen method. This ratio is a pure price-relation and must not reflect anything whatever due to change of standard or quality. Secondly, if economic importance is measured by the ratio of money-value of a given commodity to the money-value of all commodities, such an importance-measure is forever changing. If, however, the measure is that of some particular date taken as the basic date, then the result obtained is exactly that given by the aggregate-value method, and the algebraic expression exhibiting this shows immediately that the ratio found in any other manner does not in general agree.[1] It is well to bear in

$$\Sigma\left\{\frac{p'}{p}\cdot\frac{pq}{\Sigma(pq)}\right\}=\Sigma(p'q)/\Sigma(pq).$$

mind that by using price-ratios in computing a price-index the confusion arising from the fact that a change of the regimen affects the result is not avoided. It is merely kept out of sight. This is one of the reasons why the price-ratio method is not a desirable method. By using a definite regimen one obtains an obvious and unequivocal index, on the assumption, of course, that the regimen applies with sufficient precision to the usage of the community concerned at the two or more dates compared.

The position already indicated may well be reaffirmed, and the matter clearly set out as follows:

(a) If indexes are to be used for measuring the change in the relationship between money and commodities, obviously the scheme of measurement ought not to be vitiated by incorporating in it elements due, not to change in this relationship, but to differences in the character of the composite-unit by means of which the change is really

[1] For if the price-ratio t' is p'/p and q is the quantity when the price is p, the economic importance or weight of commodity x, is $p_x q_x/\Sigma(pq)$. That the sum $\Sigma\left\{r'\cdot pq/\Sigma(pq)\right\}$ is $\Sigma(p'q)/\Sigma(pq)$ is instantly seen on writing its equivalent

measured, whether the ratio of aggregate-values or the price-ratio method be followed.

(b) So long as the composite-unit is sufficiently comprehensive, and represents with adequate accuracy the custom or average usage of the community to which it is to be applied, no better measure exists for ascertaining the change in the relationship between money and commodities.

(c) The calculation of an index always supposes that the usage at the two dates may be assumed to be identical without material error, and no other assumption is equally valid.

We now return to the question of what sort of continuity is possible in price-indexes.

19. Systematic and Irregular Changes of Quantities and Prices. When indexes relate to dates widely separated in time, and also when times are abnormal, the relationship of quantities and prices do not—as a rule—hover closely about an average. They may either vary irregularly and be widely scattered about a mean, or they may show a definite trend; *e. g.*, when quantities of production are very irregular, or when the prices of commodities vary greatly or are either continually increasing or are continually diminishing. In such cases the means of the various p's and q's may be very different from those of any particular date, so much so that they form a very indifferent basis or ground for comparisons, or for obtaining an index of precise or unequivocal significance. The question arises, "Is there any real solution of this difficulty?"

It becomes necessary to inquire what indexes mean in cases of this kind, and also whether a wide "scatter" about their average value reduces their significance. In short, the question may be asked: "Are indexes less and less definite in their significance, as their scedasticity increases, or as the data from which they are computed exhibit fundamental changes?"

Let (i) $A, B, C, D, E,$ etc., denote a series of commodities;

(ii) $a, b, c, d, e,$ etc., denote another series approximately the same;

(iii) $A, B, C, D, E,$ etc., a third series, wholly different.

If the prices, and also the fluctuations of price of the three members (vertically) were sensibly identical in each case, obviously, so far as questions of exchange-value are concerned, one commodity might be substituted for any other in any one of the series.[1] This shows that

[1] For example: Suppose that coffee in one community, or at one time, was substituted for tea in another community, or at another time, and that the fluctuations in price were (sensibly) identical for the two commodities or two periods, they could still be compared, in fact treated as if actually identical. Such changes as these do not really vitiate the computed indexes. This matter is systematically considered hereinafter.

for the purpose of the computation of indexes, identity need not be absolute identity: but only identity from an appropriate point of view.

We may therefore say that in practical examples there is often what may be called a vicarious equivalence of commodities, a point which we shall more fully consider later. But before doing so we notice that certain types of changes have very little effect. Let the consideration of the matter be restricted to relatively minor variations in quantities and prices, and note that if, for two different series of commodities, the fluctuations of price-indexes, or quantity-indexes were identical, then any lack of identity in the quantities in the regimen or in the series of prices—as the case may be—would be immaterial, so far only, however, as the two series in question are concerned. From this it is self-evident that when such any identity is only approximately fulfilled, the observation can apply only approximately. Moreover, be it noted, that if the ratio of the prices for a given commodity, for any two dates in question, is in the ratio of the deduced price-index, the inclusion or omission of that commodity in the regimen will not affect the value of the index. A like proposition applies with regard to quantities.[1]

A variation in the price (or in the quantity) of an element in a regimen affects the deduced index, in the measure which the ratio of its product into the quantity, bears to the total to which it belongs. That is if p_1 becomes $p_1+\delta p_1$, then the index changes to $I\{1+q\delta p_1/\Sigma(p_1q)$. It is thus easy to measure the effect on the index of any uncertainty in a price. If a quantity be uncertain it has less effect because it appears in both numerator and denominator. Thus if it were $q+\delta q$ the effect is very approximately to change I into $I\{1+p_1\cdot\delta q/\Sigma(p_1q)-p_0\cdot\delta q/\Sigma(p_0q)$, a relatively small quantity. A very little consideration will show that the values of indexes do not depend upon very exact determinations of the quantities q_a, q_b, q_c, etc. They can be changed quite appreciable amounts without materially affecting the index, a fact which has sometimes been lost sight of, when attention is fixed upon either aggregate $\Sigma(pq)$, instead of upon their ratio. We thus see that even with a changed regimen the comparability of price-indexes if often better than might at first be expected, owing to the vicarious equivalence of commodities, and to the fact that, for price-

[1] If the Italic letters denote quantities and the Greek letters prices, the two cases are:

$$I=\frac{a\alpha_1+b\beta_1+c\gamma_1+\ \cdot\ \cdot\ \cdot}{a\alpha_0+b\beta_0+c\gamma_0+\ \cdot\ \cdot\ \cdot};\quad Q=\frac{a_1\alpha+b_1\beta+c_1\gamma+\ \cdot\ \cdot\ \cdot}{a_0\alpha+b_0\beta+c_0\gamma+\ \cdot\ \cdot\ \cdot}.$$

If it happened that say $b\beta_1/b\beta_0=I$, then the omission of this item from the regimen would itself have no effect on that index. Similarly if an additional term, say $e_1\epsilon/e_0\epsilon=Q$, the addition of the item to regimen would not of itself directly affect the value of Q. Indirectly these changes would have an effect because they alter the aggregate values and thus cause other variations to become relatively larger or smaller as the case may be.

indexes, quantities need not be ascertained with meticulous precision; and similarly for quantity-indexes prices need only be approximate.

Turning now to the consideration of the larger changes, when one adopts the pseudo-continuous scheme of Section 16, the result, putting $n=m+1$, is:

$$(17) \quad . \; . \; . \quad I_{0n}=\frac{\Sigma(p_1 \cdot q_{01})}{\Sigma(p_0 \cdot q_{01})} \cdot \frac{\Sigma(p_2 \cdot q_{12})}{\Sigma(p_1 \cdot q_{12})} \quad . \; . \; . \; \frac{\Sigma(p_n \cdot q_{mn})}{\Sigma(p_m \cdot q_{mn})} \equiv I_{01} \cdot I_{12} \cdot I_{23} \quad . \; . \; . \quad I_{mn}.$$

It is obvious that if the quantities q_{01} are maintained throughout I_0^u is both $\Pi\,(I)$ and $\Sigma(p_n \cdot q_{01})/\Sigma(p_0 q_{01})$; and that otherwise it is not so (excepting fortuitously): in short that

$$(17a) \quad . \; . \; . \quad I_{0n}=\Pi(I) \text{ is not } I'_{0n} \equiv \Sigma(p_n \cdot q_{0n})/\Sigma(p_0 \cdot q_{0n}).$$

It is also evident that the less systematic change there is in the values of q, the more nearly is the condition of real continuity fulfilled. Also while it is satisfactory to regard this final expression for I'_{0n} as the best index based on the prices of the terminal dates, they are out of real relation because the quantities q_{0n} do not substantially apply to either date; initial or terminal. This is the crux of the whole matter. Reality and definitiveness appertain to price-indexes only in so far as it is possible to deduce them upon a common basis. The appropriateness of their applicability otherwise is limited. The fluctuation of indexes, for example, for a rice-fish-eating community do not apply to a bread-meat-eating community, though by a stretch of imagination it is supposable that the community-purchasing efficiency of money in the two cases might move in agreement. This will be adverted to later. We return to the question of vicarious equivalence.

(To be concluded)

NOTES

THE SUPERVISING OR STAFF STATISTICIAN[1]

By Donald R. Belcher

Any thoroughgoing search for homogeneity and logical unity in the so-called science of statistics soon leads to the conclusion that statistics is no science at all, but rather a method—a method common to all sciences in which the observation and ordered classification of facts have an essential place. These sciences are exceedingly diversified, with the result that, not only does the general public fail altogether to grasp the comprehensiveness of statistical method in its application, but even its warmest and most professional devotees are apt often to mistrust its use in fields foreign to those with which they themselves happen to be associated. Furthermore, the statistical method itself extends over a vast range of doctrine from the simplest principles governing the collection of data to the most intricate problems in the calculus of probabilities. Is it any wonder, then, that the profession of "Statistician" has never been, and perhaps never will be, well defined?

Your committee has asked me to discuss the supervising or staff statistician—what he should know and where and how his training should be given. If it is difficult to define "statistician," it is well-nigh impossible to arrive at any sharp delineation of the staff statistician as contrasted with his superior—the executive statistician—and his subordinates—the statistical assistants. In many cases the difference is one largely of maturity and breadth of experience, so that the statistical assistant of yesterday becomes the staff statistician of today and the executive statistician of tomorrow. However, these cases probably represent the exceptions and we commonly find, especially as between the staff statistician and his assistants, essential differences in kind rather than in degree.

The staff statistician, as I have him in mind, is placed directly in charge of specific statistical investigations. He is responsible, under his executive, for the detailed planning and administration of these investigations, for the collection and tabulation of necessary data, for their analysis by appropriate methods, for the application of that

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incisive and sequential thinking which will lead to results of practical usefulness, and for the adequate presentation of those results. To accomplish all of this, he must ordinarily have an extensive and personal acquaintance with the operations of the particular enterprise by which he is employed. He must have knowledge of those phases of statistical technique which are applicable to his particular field and, while he is not necessarily himself a technician, it may well happen that he will require the services of such men on his staff and, accordingly, must exercise responsible supervision over their work. He must have the faculty for administration. Furthermore, he must combine knowledge of the business, knowledge of statistical method and administrative faculty with the ability to transform his theoretical results into recommendations for practical action. Whatever the validity of the statistical results of his investigation, they are of value to his organization only in so far as he is able to make effective presentation of them to his executive and through him to the officers who have final decision as to company policies and operation. This requires that he be able to make resourceful and effective use of graphic methods, that he be able to write lucid and forceful English and that, when opportunity arises, he be able to discuss and defend his conclusions in such a way as to instill confidence and carry conviction. But over and above all of these requirements, he must be possessed of that disciplined imagination by which, first of all, long before his data are even collected, he may visualize the entire result and its probable conclusions in order to determine in advance whether those conclusions will be of such practical value as to justify the time and money cost of the proposed investigation. He must have that disciplined imagination by which, having collected his facts, exactly and impartially, he may recognize their significance and arrive at a scientific and unbiased judgment. I have said that the staff statistician is placed in charge of specific statistical investigations. That is true in a measure, but to fulfill his complete function he must search constantly for new fields in which to make application of his statistical method. This requires, again, not only knowledge of the problems and aims of the business in which he is engaged, but the possession of imagination, disciplined and constructive.

How then may we secure men with these qualifications? Shall a business organization employ an already trained statistical technician and teach him the details of its particular business or, selecting a man already in the business who is acquainted with its operations, teach him or have him taught the statistical method? For the large and well balanced statistical organization both procedures are probably

desirable. There will normally be included among its staff statisticians men who are primarily technicians and have only a limited knowledge of the business, as well as men of long experience in the business who have analytical and administrative ability without extensive training in technique. In so far as the business organization requires the services of statistical technicians, it must obviously, in general, look to the colleges and universities to supply them. There is undoubtedly an excellent opportunity in the business world for the statistical technician who comes out of college with little practical knowledge of the problems and operations of business. The business organization will take him at that stage in his training and thereafter teach him such details of its particular business as are essential to his job.

However, it is easy to overemphasize, and I believe there is a definite tendency on the part of many teaching statisticians to overemphasize, the purely technical aspect of statistical analysis. In the work of staff statisticians there is probably a greater need for, and in so far as advancement to high executive rank is concerned greater opportunity for, the non-technician who possesses the educational and mental qualifications which I have described. This man, while ordinarily a college graduate, is very frequently taken into the statistical department only after he has had considerable experience elsewhere in the organization. He will undoubtedly receive help from properly written textbooks on statistics and possibly from attendance at collegiate courses in statistics if these happen to be available to him; but he will ordinarily not aim to become a technician and will rely instead upon technicians already in his department. It does not follow, however, that the opportunity offered the colleges and universities for the training of these non-technicians is small. On the contrary, it is exceedingly great. The business world has come to depend almost wholly upon the institutions of higher education to supply the men in whom those habits and qualities of mind—which are essential to statistical analysis in its most valuable form—have been developed. In this respect I believe that, with certain notable exceptions, the teaching statisticians of our colleges and universities have failed to recognize their responsibilities in the training of future statisticians.

For the past several years it has been my personal experience to interview numerous young men who have recently been, or are about to be, graduated from some one of the universities and colleges. These men, in general, have taken one or two courses in statistics in the confident belief that they have thereby fitted themselves for statistical work in the field of business. But a very large majority of these men

have had no mathematics beyond that of the preparatory school or possibly a Freshman course in college, no modern science other than the purely descriptive courses, and no economics worthy of the name. Do these men know statistical method? Are they equipped for successful statistical analysis in the field of business? My judgment is that they do not know statistical method and that they are not equipped for statistical analysis, in spite of the fact that they may know something about tabulation, averages and the measurement of dispersion. In our training of men for the statistical field, we must see far more clearly than we have in the past that to the practicing statistician the statistical method is never an end in itself but only a means to an end, that genuine analytical ability is not likely to be developed through a study merely of the formulae and technique of statistics, but that recourse must be had, in particular, to the continued and intensive study of the mathematical and physical sciences.

It must not be thought that training in the mathematical and physical sciences is desirable only because of the informational content of these courses. The business statistician may never have occasion to use the formulae of the differential and integral calculus or to make application of his knowledge of chemical reactions and the modern theory of relativity. But let us remember that throughout the realm of human thought there is no other field than mathematics which presents so great an opportunity for logical and rigorous thinking and the development of disciplined imagination. Here we may find truth—absolute, exact and invariant. Modern science—and I have particularly in mind physics, chemistry and biology—modern science has for its end and aim "the classification of facts, the recognition of their sequence and relative significance" and the formulation of sound and unbiased judgment. In short, the mathematical and physical sciences are characterized by the very type of analytical and sequential thinking for which the non-technical, as well as the technical, statistician has need.

In a recent address, Professor R. A. Millikan made the following statement which is apropos of our present discussion:

> With the gradual disappearance of the classics and the rigid discipline which they have furnished as the basis of our higher educational system, there have been slowly creeping into it during the past two decades certain emasculating influences which need to be counteracted. There is no Elisha upon whom the mantle of the classics can fall except the mathematical and physical sciences. There is no training like that which they furnish for teaching men to apply themselves intensively, to observe carefully and correctly, to treat their data honestly and dispassionately, and to reason objectively from a given set of conditions to their inevitable consequences —in a word, to see clearly and to think straight.

I submit, then, that the job of the statistician is to observe carefully and correctly, to treat his data honestly and dispassionately, to reason objectively from a given set of conditions to their inevitable consequences; that nowhere else in his academic experience can he find so excellent a field for development as in the mathematical and physical sciences. This fact will some day, I hope and believe, receive adequate recognition on the part of those men who are responsible for statistical training in the colleges. When that time arrives, we shall see statistical training characterized by a scientific dignity commensurate with the needs of the modern business world.

A SIMPLIFIED METHOD OF DETERMINING WHAT CONTRASTS BETWEEN TWO PERCENTAGE DISTRIBUTIONS ARE SIGNIFICANT

By Hornell Hart, *University of Iowa*

Educational and biological statistics make use of the probable error of the coefficient of correlation to determine whether a relationship between associated variables is significant or is merely due to chance. Social statistics, however, rely chiefly on comparisons between averages and percentages as indices of association between characteristics whose causal relationships it is desired to investigate; and the common practice has been to fail to apply any accurate tests to determine whether observed contrasts are likely to be due to other causes than chance. The reason for this failure in social statistics seems to be that the formulae required are unfamiliar and rather forbidding, and that when a considerable variety of characteristics of two contrasted groups are compared, the labor of applying the formulae is very considerable.

An example of the danger of such negligence is to be found in an article which compares 300 parole violators with 300 men who succeeded on parole, and with 80 men who completed their sentences without being paroled, from the Massachusetts Reformatory.[1] The author of that study, after laborious tabulation of data and calculation of the percentage distributions of the three groups with respect to the answers to each of 60 different questions on which information relating to prisoners is at the disposal of the parole board, concludes that "the Board would not improve matters by considering any of

[1] Warner, Sam. B. "Factors Determining Parole from the Massachusetts Reformatory." *Journal of Criminal Law and Criminology.* Vol. 14 (1923) pp. 172–207.

the sixty-odd pieces of information placed at its disposal, which it now ignores."[1] The reason in the mind of that investigator for discarding these items of information appears to be that he does not consider the observed differences to be statistically significant. He says: "No considerable difference between the columns (of percentages) appears."[2] Among the items discarded is the fact that 15 per cent of the men who violated their paroles had reported that their fathers had served jail sentences, while only one per cent of the men who succeeded on parole reported their fathers as having been in jail.[3] Of all the men paroled, 8 per cent so reported their fathers. Applying Yule's formula[4] to this case:

$$\epsilon^2_{12} = p_0 q_0 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)$$

the standard difference (ϵ) in this case is 2.2 per cent, while the observed difference is 14 per cent, or over six times as large. Such a difference would be exceeded by chance only once in millions of trials. Of the 43 questions discarded as not affording significantly different answers for the two groups, 14, or one-third showed contrasts in the answers greater than would tend to be exceeded by chance once in a hundred times.

The labor of applying the formula to all of the answers to all of the questions in such a study as that referred to, might well be a deterrent. The purpose of the present article is to present a simplified method for applying this criterion very rapidly to all the observed differences in such a study.

Strictly speaking, in such a study as this, we are interested in the differences in the percentages of parole violators among the various classifications of paroled men rather than in the differences between violators and non-violators with respect to the other characteristics. The former sort of inquiry, however, would involve dealing with samples of widely differing sizes, while the numbers of parole violators and of non-violators remain constant throughout the study. It is economical, therefore, to substitute the second sort of inquiry for the first, since the conclusions reached will be the same in either case. This makes n_1 and n_2 the same for each solution of Yule's formula in the study in question. The value of ϵ for the entire investigation thus becomes a simple function of q and may be plotted readily on coördinate paper.

[1] *Op. cit.* p. 196.
[2] *Op. cit.* p. 190.
[3] *Op. cit.* p. 198.
[4] Yule, G. Udney. *Theory of Statistics.* London, 1919. P. 269.

Yule's formula gives the standard error of the differences between the percentages having the given characteristic in the two samples. It is customary to multiply standard errors by .6745 in order to get

Table 1

p or q	q or p	$\sqrt{pq}$	$\frac{2.5757}{\sqrt{pq}}$	p or q	q or p	$\sqrt{pq}$	$\frac{2.5757}{\sqrt{pq}}$
.00	1.00	.0000	.0000	.25	.75	.4330	1.1153
.01	.99	.0995	.2563	.26	.74	.4386	1.1298
.02	.98	.1400	.3606	.27	.73	.4440	1.1435
.03	.97	.1706	.4394	.28	.72	.4490	1.1565
.04	.96	.1960	.5047	.29	.71	.4538	1.1688
.05	.95	.2179	.5614	.30	.70	.4583	1.1803
.06	.94	.2375	.6117	.31	.69	.4625	1.1913
.07	.93	.2551	.6572	.32	.68	.4665	1.2015
.08	.92	.2713	.6988	.33	.67	.4702	1.2111
.09	.91	.2862	.7371	.34	.66	.4737	1.2201
.10	.90	.3000	.7727	.35	.65	.4770	1.2285
.11	.89	.3129	.8059	.36	.64	.4800	1.2363
.12	.88	.3250	.8370	.37	.63	.4828	1.2436
.13	.87	.3363	.8662	.38	.62	.4854	1.2503
.14	.86	.3470	.8937	.39	.61	.4877	1.2563
.15	.85	.3571	.9197	.40	.60	.4899	1.2618
.16	.84	.3666	.9443	.41	.59	.4918	1.2668
.17	.83	.3756	.9675	.42	.58	.4936	1.2713
.18	.82	.3842	.9896	.43	.57	.4951	1.2752
.19	.81	.3923	1.0105	.44	.56	.4964	1.2786
.20	.80	.4000	1.0303	.45	.55	.4975	1.2814
.21	.79	.4073	1.0491	.46	.54	.4984	1.2837
.22	.78	.4142	1.0670	.47	.53	.4991	1.2855
.23	.77	.4208	1.0839	.48	.52	.4996	1.2868
.24	.76	.4271	1.1000	.49	.51	.4999	1.2876
				.50	.50	.5000	1.2879

the "probable error." I prefer to multiply the standard error by 2.576, which gives the error likely, in a normal distribution of errors, to be exceeded by chance once per 100 pairs of samples.

The justification of such a procedure depends, of course, upon the normality of the distribution of the errors. As a test of this assumption the case was investigated of pairs of samples drawn from an infinite universe in which p_0 is .1, q_0 is .9, n_1 is 20, and n_2 is 100. The observed differences between p_1 and p_2 in this case fall into a slightly

skewed frequency curve, with a standard deviation of 7.337. The probable error, according to the usual formula, would be .6745×7.337, or 4.949. The distribution of differences is, of course, discontinuous; but 45.54 per cent of the differences fall between plus 4.5 and minus 4.5, while 54.22 per cent of the cases fall between plus 5.5 and minus 5.5. In a normal curve one per cent of the cases lie farther than 2.576 standard deviations from the mean. In the distribution in question 2.576×7.337 is 18.90; 1.27 per cent of the differences lie beyond plus 18.5 and minus 18.5, while 0.94 per cent lie beyond plus 19.5 and minus 19.5. Even with n_1 equal to only 20 cases with n_1 only one-fifth as large as n_2 and with p equal to .1, wherefore, the distribution of differences approaches very closely to the normal curve. Where n_1 equals n_2 equals 300 the distribution should be almost exactly normal.

Values of 2.58 pq are given in Table I. In the present problem, for the comparison between parole violators and non-violators, n_1 and n_2 are both 300; $\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ therefore equals .082. Comparison of men paroled with men not paroled is also desirable. For this purpose n_1 is 600 and n_2 is 80; $\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ thus becomes .119 for this comparison. The values of ϵ are plotted for various values of p_0 in Chart 1. The use of this chart may be illustrated by the case already cited. Of all the paroled men 8 per cent reported that their fathers had been in jail. Looking up 8 per cent on the 300–300 line, the value of ϵ is found to be 5.6. In other words, if pairs of samples were drawn from an unlimited universe in which 8 per cent had this characteristic, the differences in the percentages of this characteristic in the pairs of samples would tend on the average to exceed 5.6 once in 100 pairs of samples. The observed difference is 14 per cent (*i. e.*, between 13.5 and 14.5), which is at least 2.4 times as large as ϵ. Assuming a normal distribution of the differences, such a difference would not be exceeded by chance oftener than once in a billion trials.

While the assumption of a normal distribution of differences may at times be slightly in error, it is desirable to have some way of estimating not only whether a given difference would be exceeded by chance once in 100 trials, but also about what is the probability of its being exceeded. Chart 2 shows on semi-logarithmic paper the probabilities of various multiples of the errors being exceeded by chance. The scale on the left shows the difference which would be exceeded by chance once in 100 trials (2.567 times the standard dif-

ference). Two times this unit, it will be seen from the chart, would be likely to be exceeded about twice in 10,000,000 trials, under a normal distribution. For convenience a scale in terms of the probable error is given on the right.

By the use of Table I charts similar in principle to Chart 1 may be

Chart 1.

Values of $\epsilon = 2.576\sqrt{pq\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$

where $q = 1 - p$, for the values n_1 300; n_2 300
and for the values n_1 600; n_2 80

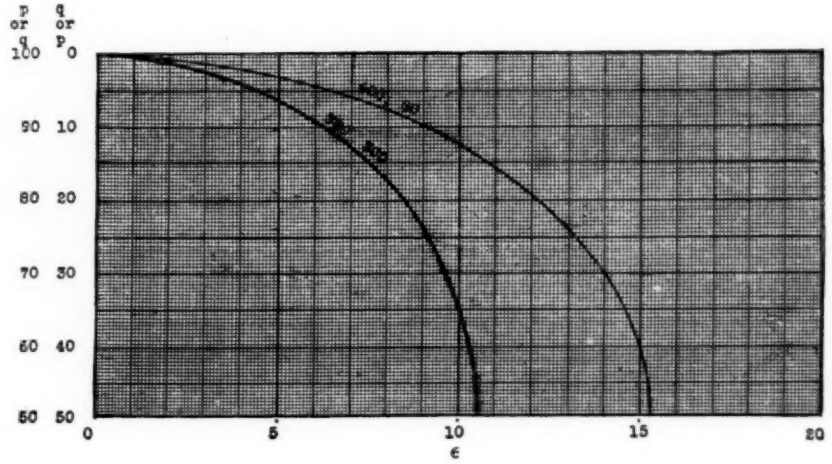
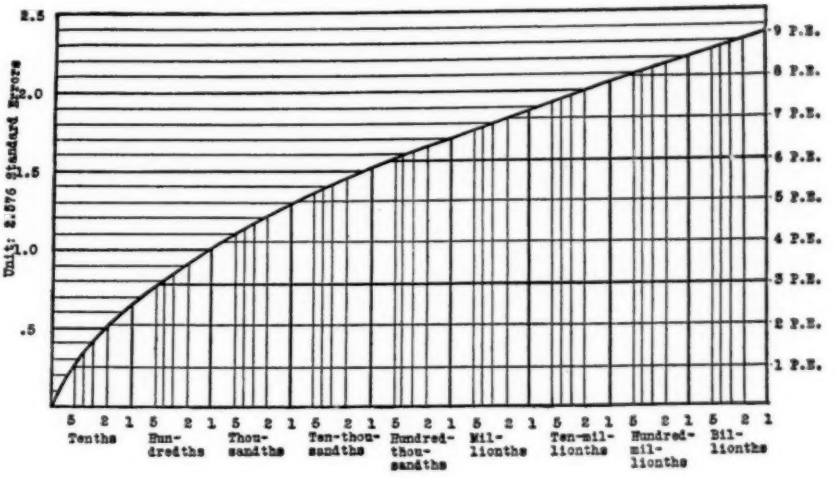


Chart 2.

Chart for determining the Probability
that a Given Result is Due to Chance
Under the assumption that the errors follow the normal frequency curve



constructed for any investigation involving comparisons between percentage distributions of two or more contrasted groups. Calculating and drawing the chart requires only a few minutes, and after it is done the significance of the various observed differences in percentage distributions of the contrasted groups may be tested almost instantaneously.

THE POLITICAL VOTE AS A FREQUENCY DISTRIBUTION OF OPINION

By Stuart A. Rice, *Dartmouth College*

Variability is an outstanding characteristic of social data. Whether we are measuring such physical characteristics as height, weight or chest expansion, or whether we are concerned with intellectual capacities or performances, a sufficiently large number of measurements upon homogeneous individuals seems to point invariably to a *continuous* distribution from the lowest measures to the highest, with a massing of cases at some point on the scale which is usually about mid-way between the two extremes.

It is assumed that if the number of individual cases could become infinite, every position on the scale, however small the gradations, would be represented by frequencies. In practice, frequencies are assembled together during classification around *mid-points*, so that a distribution which is theoretically continuous is actually presented in any given series as if it were discontinuous or discreet. The measurements given *approximate* the actual dimensions. This process of classification, it should be noted, is a wholly artificial one in the sense that the number of classes, together with the location of mid-points or class limits, depend entirely upon the will of the classifier, who is presumably guided by the purposes for which the data are to be employed.

There is no reason to suppose that the political opinions held by individuals do not follow the normal frequency distribution which is characteristic of more easily measurable mental characteristics or products. It seems to be true that opinions might be classified in many ways, none of them measurable at present with any degree of exactitude. Nevertheless we are accustomed to distinguish between opinions on either side of a given issue which are lightly or strongly held, between those which are predominantly emotional and those

which are predominantly intellectual in character, between those which are moderate and those which are extreme. Thus we recognize variability, and in several directions, but we neglect to note the intermediate positions along these various scales of opinion, because we are unable to do so. Moreover, the democratic concept, in which, according to Bryce,[1] the idea of "one man one vote" is central, has tended to direct attention away from qualitative differences between individual opinions.

It is conceivable that with the development of the technique of mental measurements, qualitative differences in individual opinions might be valued in the process of measuring the collective decision on a given political issue. With respect to the collective decision on prohibition, for example, we might assign greater value to the opinion which was based on scientific analysis of the physiological, social or economic effects of the liquor traffic than to the opinion which represented prejudice, self-interest or casual impression. We might even be willing to weight the opinion which represented strong conviction, relative to the opinion which was weakly held and very near the point of indifference. According to any one of the possible lines of classification, an average opinion might theoretically be obtained, in which each opinion was weighted according to its place along the scale in the entire distribution.[2]

Modern political society, however, resorts to a crude measurement of opinion by means of the *vote*. With this device, opinions which would normally be distributed continuously are consolidated into a discreet series, containing but two classes. That is, opinions which would normally fall at all values along the X ordinate are grouped into two classes of *pro* and *con*. Instead of smoothing the histogram into a curve, the reverse process occurs: out of the curve is constructed a histogram of two classes containing all of the frequencies in the series.

The class limits, as in the case of every other series, are fixed arbitrarily; but since there are but two classes, each extends in one direction to the extreme value included in its side of the distribution. Hence there is but one class limit to be arbitrarily defined—the point at which the issue shall be drawn for presentation to the voters. According to the location of this limit on one side or the other of the mode of opinion, the verdict of the electorate will be "for" or "against."

[1] Bryce: *Modern Democracies*, pp. 20–23.

[2] An approximation of this idea is contained in the system of plural voting which is based upon property ownership. Where such a system exists, *any* opinion held by the plural voter is accorded greater value than the corresponding opinion of the citizen with a single vote. The hypothetical system here suggested would grade the *opinions*, rather than the persons who held them.

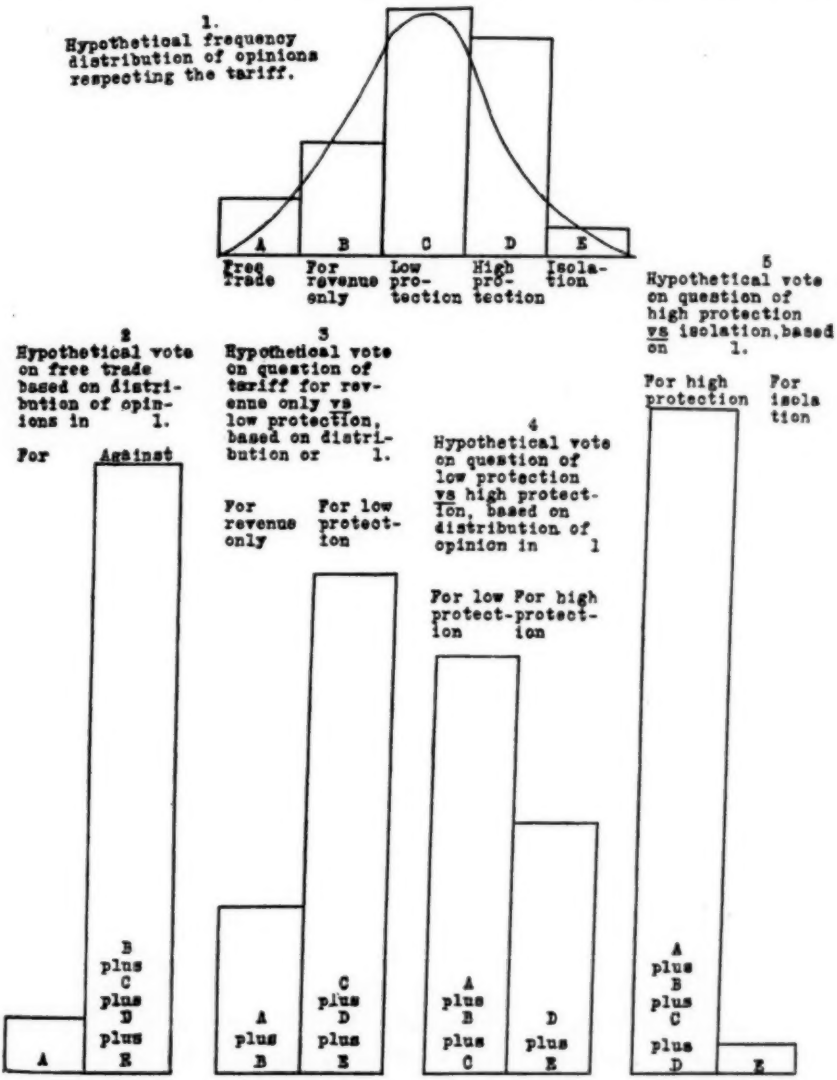
Statesmen and politicians are successful according to their ability to estimate this modal opinion and define the issues accordingly. It follows that no genuine issue is likely to remain drawn at a point which is far removed from the central massing of opinion. Extremists, sensing the futility of getting their views adopted, or even voted upon, decry political methods and call for "direct action" to secure their ends.

The hypothesis that has been presented may be tested by its application to some leading political issues now before the American people. Among these we may first consider the tariff. It is possible to distinguish five positions in order along a scale which would represent differing views upon this question. These views would be (1) absolute free trade; (2) tariff for revenue only; (3) low protection; (4) high protection; (5) complete isolation, or embargo on imports. These views may be represented graphically in Figure 1. That each position is relative and shades imperceptibly into the position adjoining, will be apparent if we consider the differences between high protectionists and low protectionists.

Let us assume a legislative body, each member of which is in favor of a tariff, but uninfluenced in his vote by party considerations or factors other than his own opinions. Assume further that this legislature is confronted with the task of fixing the duty upon a specified commodity. Suppose that proposals for an ad-valorem duty are brought before this body, each proposal in turn representing an increase of 5 per cent in the tariff contemplated. We should expect with each proposal to find a number of legislators who would desert the high tariff side of the division and go over to the low tariff side. Thus the number of high protectionists and low protectionists, so far as the tariff is judged upon its merits alone, depends entirely on the point at which the issue is drawn for decision.

In the American Congress the tariff issue has usually been drawn within the ranges of opinion which we have characterized as "low protection" and "high protection." The Republican and Democratic parties, as represented in Congress, have stood for "higher" and "lower" tariffs, respectively. It is obvious that American opinion, as distributed between free trade on one extreme and national economic isolation on the other, tends to mass within a range calling for some kind of tariff protection for American industries. Too great a departure from this central mass of opinion in either direction by the party in power will throw the mode of the distribution of opinion into the class represented by the opposition party, and create a party turnover at the next election.

In Figures 2 to 5 inclusive the frequency distribution of opinion respecting the tariff which was postulated graphically in Figure 1, has been broken up into a series of bi-segmented histograms to rep-



resent the probable result of a vote, if the issue were drawn in turn between each two consecutive positions of the five that were distinguished. Thus if the vote were upon the question of free trade (Figure 2) we should expect the persons whose opinions were represented

by A (in Figure 1) to vote in the affirmative, and those represented by B, C, D and E to unite in the negative. If the vote were upon the question of low protection *vs.* high protection (Figure 4) we should expect A, B, and C to unite in the affirmtive against D and E.

The continuous distribution of opinion on another question was indicated by the notable poll on prohibition taken by *The Literary Digest.*[1] The histogram of opinion, in this case, contained three classes rather than the conventional two presented by an "aye" and "nay" vote. The voter could be recorded either (1) for continuance and strict enforcement of the Eighteenth Amendment and Volstead Law; (2) for modification of the Volstead Law to permit light wines and beers; (3) for repeal of the Prohibition Amendment. The fact that no one of these classes of opinion was represented by a majority of the votes led to claims by the propagandists on both sides that the results represented a victory for their own extreme point of view. "Drys" called attention to the majority against a return to "wet" days. "Wets" called attention to the majority against the existing stringent enforcement laws. Each drew the issue at a different point along the opinion scale.

As still a third distribution of opinion, we may cite the various positions taken upon the question of public ownership. At one extreme the anarchist holds for complete individualism, at the other the extreme socialist would place all industry under state control. In the middle ranges of opinion are to be found those who would nationalize the railways; those who would add coal mines to railways; those who would add to these the packing plants and the flour mills. It is probable that individual and class interests help to determine the particular industries or services which the individual would be willing to have publicly owned. Hence as to any particular industry, opinion might be multi-modal or skewed. It remains probable that the thoroughgoingness of public-ownership opinion in the abstract remains normal in distribution.

If political opinions are distributed in the normal manner that we have suggested, it is probable that radical changes in public opinion occur less frequently than is usually supposed. The relative strength of parties at election time may be changed in either of two ways: In the first, the points at which the issues are drawn may remain constant. Hence, a comparatively slight shift of the modal opinion may bring about a transfer of power from one party to another that appears superficially as a "sweeping verdict at the polls." Nevertheless, the shift is most likely to have taken place within the central

[1] *Literary Digest*, July 8, 1922. Final returns, Sept. 9, 1922.

quartiles of the distribution where opinion most nearly approaches indifference. Subsequent events usually prove in such a case that the changes of opinion were not profound or thoroughgoing. Opinions lightly held are easily changed. Thus the fickleness of public opinion that is so frequently observed may be a phenomenon representing the comparative indifference of the central portions of the distribution of opinion.

In the second case, opinions may remain distributed in the same way, but the points at which the issues are drawn may be shifted. When this occurs, it is usually the result of manouvering for advantage on the part of politicians and party leaders. Old issues are presented in a new light, so that in effect the opinions of the electorate are reclassified and new class limits are established. Any shift of party strength between two elections in which the same issues are presented probably involves both of these occurrences.

There remain to be pointed out the implications of the hypothesis upon movements for political reform. These are more likely to succeed when proceeding step by step, than when presenting their complete program in toto. The mode may be shifted, but slowly. If the political change which is made as a result of the first distribution of opinion proves satisfactory to the voters, the modal shift is likely to continue in the same direction, and successive steps may prove possible.

For example, a distribution of opinion three decades ago with regard to the political rights of women would probably have disclosed the following points along the scale: (1) Women should have no voice in public matters; (2) women should have a voice in voluntaristic organizations, as in church societies, but in no other; (3) women should have the suffrage with respect to school elections; (4) school and municipal suffrage should be given; (5) all suffrage except for national elections should be allowed; (6) complete national suffrage as well as local; (7) women should unite in politics, attempting to secure sex dominance in public matters. With many minor variations, the woman suffrage issue has been drawn in turn between each consecutive pair of these various positions on the scale. The mode of opinion has been shifted constantly in the same direction until it is now to be found, quite clearly, within the group of opinions which sanction all forms of *voting* equality between men and women. It has not yet shifted to the point at which *office-holding* for women is regarded with equal favor, nor to the point at which woman's dominance in politics can be regarded as a practical possibility.

ANNUAL REPORT OF THE COMMITTEE OF THE AMERICAN STATISTICAL ASSOCIATION ON GOVERNMENTAL LABOR STATISTICS

Following the last annual meeting at which the committee made a full report on employment statistics, it was continued by the Association as a Committee on Governmental Labor Statistics, with the enlarged scope which the new name implies. During the year, therefore, we have begun work on plans for the more adequate collection of wage statistics by governmental bureaus. The attention of the committee, however, has been devoted chiefly to employment statistics, since much remains to be done to make them complete enough to give an accurate picture of the trends of employment in the United States.

By vote of the committee, this report relates to the collection of employment statistics by the federal government, and contains our recommendations as to the kind and scope of data needed, and the bureaus which, we believe, should be held responsible for the work.

We would, first, remind the members of the Association that this committee was appointed to coöperate with the Committee on the Business Cycle, which was appointed by Secretary Hoover, following the President's Conference on Unemployment. The Committee on the Business Cycle, whose report was published last spring, urged the importance of statistics of employment for the two-fold purpose of showing (1) trends of employment and unemployment as a social condition affecting human beings and their welfare; and (2) trends of production and business activity.

To accomplish this purpose, the committee believes that the data must be (a) periodic; (b) promptly available; (c) national in scope, making possible a national index; (d) inclusive of all important industries of the country, and published separately for them; (e) available for different sections of the country, and for important cities, to reflect possible variation in different localities; and (f) simple in form, to make possible prompt and general reporting.

Specifically in line with these general requirements, the committee recommends (a) that the data asked for be, as at present, the total number on the payroll, and the total wages paid on the pay day nearest the 15th of the month; (b) that these two facts be reported monthly; (c) that the industries included, as listed by the committee,—gauged by importance as a measure of economic activity, and feasibility of reporting,—be (1) manufacturing in its main industrial divisions, (2) mining and quarrying, (3) communication, (4) building construction, (5) wholesale trade, and (6) retail trade, (7) logging, and (8) agriculture.

It seems unnecessary to comment on these recommendations as they were fully discussed at the meeting in Chicago last December. At this time it seems to us vital that the Association should take definite action regarding a unified plan of collection, designating the bureaus in the federal government which should be responsible for the national index, their relation to each other, and to the state bureaus of labor statistics. In making these recommendations, it must be borne in mind that statistics of employment have been collected for the past ten years,

and that a feasible plan must have the sanction of this past experience, and must, also, be related to the present organization of governmental services. It seems hardly necessary to point out how great is the need for a better organization of the statistical services in the federal government than now exists, since the present confusion and duplication between departments not only cause waste but result in giving to the public fragmentary information. The Committee on Governmental Labor Statistics has adopted an opportunistic policy and makes its recommendations to fit as effectively as possible the present scheme of organization. It should be remembered that in any effort to centralize statistics in a single governmental bureau, we face the difficulty that statistics which are needed in the administrative routine of a department can, perhaps, be most effectively gathered by that department even though theoretically, as a statistical service, it belongs elsewhere. Our recommendations, the majority of which constitute a record of present practice, are as follows:

(1) As between the states and the federal government, we would put the initial responsibility for the collection of employment statistics upon each state. Each state needs to include a larger number of establishments in its own area to reflect its industrial activities than the federal government needs from the same area for a national index. It seems logical and efficient, therefore, for each state to collect the data and to send to the federal government such portion of its returns as may be needed for a national index. This plan is now in operation in New York, Wisconsin, Illinois, Massachusetts and Maryland.

(2) That the Federal Bureau of Labor Statistics continue to be the coördinating center to receive the reports from all the states now collecting these statistics, to stimulate other states to join in the plan, and, in the meantime, to collect as it now does the data from establishments not located within a state now collecting them, and that the bureau as rapidly as possible extend the scope of its employment statistics beyond manufacturing, notably to include the building trades which seem unlikely to be covered by other departments.

(3) That the Geological Survey add to the reports which it now receives about coal mining the necessary questions regarding employment and earnings in mining and quarrying and that the results be reported promptly once a month to the Bureau of Labor Statistics.

(4) That the Interstate Commerce Commission continue to send to the Bureau of Labor Statistics the data regarding employment on the railroads.

(5) That the Federal Reserve Board and Banks be urged to include statistics of employment in retail and wholesale trade in the questionnaires now received by Federal Reserve Banks from retail and wholesale trade, and to report these to the Bureau of Labor Statistics except in those states in which the bureaus of labor statistics are now collecting the data, or will in the future undertake it. In these states the state bureau of labor statistics should make a report on employment in retail and wholesale trade to the Federal Reserve Bank of the district.

(6) That the Department of Agriculture be urged to carry forward its present plans for experiment in the collection of facts about employment in agricultural districts. The committee recognizes that in agriculture it is impossible to con-

form to the same methods of reporting which are recommended for manufacturing, since payroll data are not available in agriculture. The committee urges that appropriate statistical measurements be developed which will reflect the trend of numbers employed on the farms, and that the information be reported to the Bureau of Labor Statistics and published in connection with payroll data, though not necessarily combined with it in a general index.

(7) That in general, wherever a state bureau of labor statistics has undertaken the collection of employment statistics in any industry, the federal bureau interested in that industry should make the state bureau its agent for the collection of employment statistics, and should not itself collect employment data in that area.

(8) That the Federal Bureau of Labor Statistics be the publisher of a national Bulletin of Employment in which all employment statistics collected by state or federal bureaus shall be published promptly, and that as soon as the data justify it, a comprehensive national index of employment be included in it.

(9) That the Bureau of Labor Statistics publish employment data separately for the different sections of the country, possibly adopting the boundaries of the Federal Reserve districts.

(10) The adoption of these recommendations would carry with them the equally important, though negative, counsel, that any other department or bureau of the government should refrain from the collection of employment statistics unless it can have a definite place in this unified plan. This applies to the U. S. Employment Service, which is now duplicating the work of the Federal Bureau. We recognize the need which the United States Employment Service is seeking to meet by collecting data in specified localities for the use of labor exchanges and for other purposes, but we believe that this need can better be met by so equipping the Federal Bureau of Labor Statistics as to enable it to collect and publish the data for these localities. Otherwise business establishments will inevitably be burdened with requests for the same report to be made to different agencies of the government. There are also some Federal Reserve Banks which are at present duplicating state work in the collection of employment statistics. Here again the duplication in itself indicates the increasing need for employment statistics, but the need will be much more adequately met in the long run if a unified plan is adopted and continued.

(11) These recommendations should be coupled with vigorous effort to secure for the Bureau of Labor Statistics a larger appropriation to enable it to collect and to publish employment statistics more adequately and more promptly than is now possible.

One further matter has seemed to the Committee on Governmental Labor Statistics to be of urgent importance at this time. If employment statistics are to be satisfactorily extended and standardized, statisticians in the government service must be accorded due recognition of the responsibilities and professional content of their work. The Personnel Classification Board, which is now at work classifying positions in the federal government, has made some tentative allocations which are very disquieting. Practically all the positions in the Federal Bureau of Labor Statistics have been classified in the clerical or administrative

group. To continue to regard statistical work as clerical must postpone the day of good statistical work. Therefore, the Committee on Governmental Labor Statistics earnestly hopes that the Association at this time will take action to insure recognition of the importance of the work of statisticians in the federal government.

Respectfully submitted,

MARY VAN KLEECK, *Chairman*

December 27, 1923

A. J. Altmeyer, Secretary, Industrial Commission of Wisconsin
Charles E. Baldwin, Chief Statistician, U. S. Bureau of Labor Statistics
Joseph A. Becker, Statistician, Bureau of Agricultural Economics, U. S. Department of Agriculture
William A. Berridge, Assistant Professor of Economics, Brown University
W. Randolph Burgess, Assistant Federal Reserve Agent, Federal Reserve Bank of New York
R. D. Cahn, Chief Statistician, General Advisory Board, Free Employment Offices, Illinois Department of Labor
Frederick E. Croxton, Assistant Professor of Economics, Ohio State University
J. Frederic Dewhurst, Chief, Statistical Division, Federal Reserve Bank of Philadelphia
Don D. Lescohier, Associate Professor of Economics, University of Wisconsin
Max O. Lorenz, Director, Bureau of Statistics, Interstate Commerce Commission
Royal Meeker, Secretary of Labor and Industry of the Commonwealth of Pennsylvania
Eugene B. Patton, Chief Statistician, New York Department of Labor
Roswell F. Phelps, Director, Division of Statistics, Massachusetts Department of Labor and Industries
Walter W. Stewart, Director, Division of Research and Statistics, Federal Reserve Board
Fred G. Tryon, Statistician in charge of Coal and Coke Statistics, U. S. Geological Survey
Leo Wolman, New School for Social Research, New York, N. Y.
Ralph G. Hurlin (Secretary), Director of the Department of Statistics, Russell Sage Foundation

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A NOTE ON THE CENSUS CATEGORIES, "URBAN" AND "RURAL"

Rural sociologists have called attention to the inadequacy for purposes of social research of the distinction drawn by the Census Bureau of the United States between "urban" and "rural" territory. It is contended that within the category "rural," including as it does the residents of small incorporated places as well as the farming population, important differences in social and economic status are concealed.

Striking confirmation of this contention has recently been obtained by the writer in connection with a search for evidence regarding the comparative similarity or dissimilarity of the votes of farmers and workingmen. The results of this inquiry are not of concern here.

In two states, Minnesota and Wisconsin, election returns were available on a precinct basis. This made it possible, with the aid of the Fourteenth Census, to segregate the vote cast in all incorporated places of stated sizes, and also in the area *outside of incorporated places*.[1] Below are given summaries of the vote for certain candidates in areas combined as follows: first, unincorporated territory; second, one or more groups of incorporated places of less than 2,500 population. Both of these areas are by census definition wholly *rural*.

The vote received by Henrik Shipstead in his unsuccessful campaign for Governor of Minnesota as a radical independent candidate in 1920, and in his successful race for the United States Senate as the Farmer-Labor Party nominee in 1922, was in part as follows:

VOTE RECEIVED BY HENRIK SHIPSTEAD FOR GOVERNOR OF MINNESOTA, 1920, AND FOR UNITED STATES SENATOR, 1922 *

Areas combined	Votes for Shipstead			
	1920		1922	
	Number	Per cent of total vote	Number	Per cent of total Vote
All area "rural" by census definition	166,993	38.2	183,729	46.7
Incorporated places of 1,000–2,500 population	12,362	19.1	19,873	32.0
"Rural" area outside of places of 1,000–2,500 population	154,636	41.6	163,856	49.4
Forty counties wholly "rural" by Census definition	76,223	38.6	86,054	47.0
Incorporated places of 1,000–2,500 population (57 in 1920 and 1922)	7,043	18.8	12,271	32.7
Incorporated places of less than 1,000 population (192 in 1920; 211 in 1922)	7,539	20.1	13,116	33.0
Area mainly unincorporated	61,641	50.2	60,667	57.4

* The classification in the table is for the entire state of Minnesota *except* St. Louis County. This county was separately classified in the analysis made, because of its peculiar industrial character, and because urban and rural areas could not be distinguished in the election figures. Data is derived from the Legislative Manual of Minnesota, 1921 and 1923.

It is clearly shown that in both of these elections the farmers were *radical*. In 1920, in a three-cornered contest, half or more of their votes were cast for the radical candidate. On the other hand, the small town voters were highly *conservative*. In the same election, less than one fifth of their votes were for this candidate. His vote within these "rural" villages, in fact, was proportionately less than in any grouping of incorporated places of larger size. Mr. Shipstead's strongest support and likewise his greatest opposition came from easily differentiated population groups within the present "rural" category.

In Wisconsin, we have similarly analyzed the vote cast in the Republican

[1] Some overlapping of precinct and village boundaries appears to exist in both states. But for this fact, the contrast in political opinions which is disclosed would appear even more striking.

primary of 1920 for John J. Blaine, who bore the LaFollette and the Nonpartisan League indorsements, and who was the successful candidate both in the primary and the general election. Data is taken from the Wisconsin Blue Book of 1921.

VOTE RECEIVED BY JOHN J. BLAINE FOR THE NOMINATION FOR GOVERNOR IN THE WISCONSIN REPUBLICAN PRIMARY, 1920

Areas combined	Total vote for Republican candidates	Vote for Blaine	
		Number	Per cent of total
Unincorporated area	138,667	53,846	38.8
Incorporated places of less than 2,500 population	53,654	12,852	24.0

While the differences between farmers and small town dwellers with regard to political preferences are not so striking in the Wisconsin election cited as in those of Minnesota, the same tendencies as in the latter are apparent.

The validity of the classes defined by the Census Bureau appears to depend upon the extent to which they correspond to significant modal differences within the *social environment*, itself. If we may regard a radical-conservative cleavage as significant of important social differences, then the census categories, at least in the cases cited, failed to meet the criterion named. We appear to need new categories which will permit of easy segregation and comparison of the three important types associated with the numerical concentration of population, namely, the urban dweller, the villager and the true countryman.

STUART A. RICE

STANDARDS FOR AMERICAN VITAL STATISTICIANS

Antedating somewhat the discussion of educational and professional standards by the American Statistical Association in December, 1922, the Section on Vital Statistics of the American Public Health Association appointed (October, 1921) a Committee to inquire into suitable standards of professional competence for vital statisticians. It was expressly stipulated that this Committee should work entirely in harmony with the American Statistical Association. A first report for vital statisticians was published in the December, 1922, AMERICAN JOURNAL OF PUBLIC HEALTH, and strong endorsement was given to the proposal for a standard set of reference tests. (See March, 1923, JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION.)

The American Public Health Association Committee held that there had been considerable and commendable emphasis in recent years on registration and tabulation technique. This followed from the pioneer work of Edward A. Jarvis, Francis A. Walker, W. F. Willcox, John Shaw Billings, William A. King and Cressy L. Wilbur. But there has been marked retrogression in other ways from the standards set by Farr and Babbage in England, and by Henry Brooks

Baker and Ezra M. Hunt in the United States. The Committee contended that vital statisticians, some of whom were today simply registrationists and tabulators, fell far short of their real opportunities for public service, and that the prime function of the vital statistician was to act as a forerunner, a pathfinder, and as counselor and critic to public health forces. In order to approach his work with higher aspirations, the vital statistician needed discipline in critical analysis and this involved, in the opinion of the Committee, some basic training in the arts of higher analysis and quantitative logic. Certain details, especially the mathematical discipline, of the Committee's report have been sharply criticized. But no constructive comment has been offered to show what will take the place of the analytical technique recommended by the Committee. Certainly, the uncritical presentation of simple tables, and some inconsequential arithmetic, does not constitute competent analysis!

The Committee will continue its inquiry as far as the interests of vital statistics are concerned. It will continue to coöperate with the American Statistical Association in order that the public health movement may in the future have the service of qualified analysts. The American Public Health Association's Committee is composed of the following: Raymond Pearl, Edgar Sydenstricker, Otto R. Eichel and Edwin W. Kopf (Chairman).

At the outset of its career (1907) the Section on Vital Statistics of the American Public Health Association was earnestly dedicated to the proposition that "vital statistics is the keystone of the public health arch." This statement, attributed to Dr. Franklin C. Gram, of Buffalo, should be inscribed upon the tablet over the door of every school of public health in the United States, and its significance emphasized. Vital statistics will achieve its rightful place in public health work only after it has attracted thinkers, analysts and constructive critics to its fold. Vital statistics will then have become vitalized! At the present time the Committee earnestly believes "*aegroto dum anima est, spes esse dicitur!*"

E. W. K.

PROGRESS OF WORK IN THE CENSUS BUREAU

FOURTEENTH CENSUS PUBLICATIONS

The recent publication of Volume IV on Occupations completes the eleven volumes constituting the main reports of the Fourteenth Census. Some of the volumes previously issued are already out of print and because of our insufficient printing appropriation the Bureau is not at present in a position to order new editions. The Statistical Atlas, which is essentially a collection and republication of the maps, charts, and illustrations scattered through the various census volumes, still remains to be printed. The state compendiums, corresponding to the state supplements of the 1910 census, are also not yet published. These will contain no new matter, but will bring together for convenience in a single publication for each state substantially all the census statistics relating to that state.

REPORT ON MORTALITY RATES

The Bureau has published the report on *Mortality Rates 1910–1920*, a quarto volume of 681 pages containing annual death rates from 1910 to 1920, based on revised population estimates for each state, city, and county within the registration area. It also gives 1920 death rates by country of birth of mother and cause for certain limited areas.

CENSUS MONOGRAPHS

The following census monographs are in type and ought to be printed within the next two or three months:

Mortgages on Homes. This presents the results of the special census inquiry relating to mortgages on homes other than farm homes.
Farm Tenancy in the United States, by E. A. Goldenweiser and Leon E. Truesdell.
Integration of Industrial Operations, by Willard L. Thorp.
School Attendance, by Frank A. Ross.

In place of the proposed monograph on *Children in Gainful Occupations* the Bureau will issue a bulletin on that subject in quarto form. This change of plan was made partly in order to include a reprint of the 60 quarto pages of detailed tables contained in the main Occupation volume, with the idea that it would be desirable to have all the statistics that the Bureau has tabulated regarding children in occupations brought together in one publication. This bulletin has now been released for printing.

MARRIAGE AND DIVORCE CENSUS

The Bureau is making preparation for another enumeration of marriages and divorces covering the year 1923. There have been two comprehensive compilations of this kind each covering a 20-year period, namely 1867 to 1886 and 1887 to 1906, respectively. The next compilation covered one year only, 1916. The next was that of 1922, the results of which are now being published. The precedent once established, it is probable that this inquiry will be conducted annually hereafter.

The divorce schedule has been simplified by the omissions of a number of questions which have been carried in the past but are not believed to yield results of sufficient significance to justify their repetition regularly year after year. The questions remaining include date of marriage; which party, husband or wife, was the libellant; whether case was contested; date of decree or judgment; cause for which divorced; and number of children affected by the decree. As regards marriages the number only will be reported as heretofore.

To a large extent the data for this inquiry can be obtained directly from the state reports, or from records kept at the state capitols. So far as that is not the case agents of the Bureau have to consult the records of the county courts. But it is gratifying to note that the number of states keeping records is increasing and the Bureau of the Census, naturally, is endeavoring to promote this movement.

CENSUS OF WEALTH, DEBT, AND TAXATION

The collection of data for the wealth, debt, and taxation inquiry covering the year 1922 is completed, and preliminary press summaries have been issued for each state giving the total debt and taxes for the state and for each class of political units, namely, counties, cities, and minor civil divisions, together with the aggregate assessed valuation of property. The Bureau is now issuing a series of press releases giving the estimates of wealth. Following the press releases, four summary tables are being printed in the form of leaflets or small bulletins, giving figures by states covering respectively the subjects of: (1) public debt, (2) taxes, (3) assessed valuation and tax levies, and (4) estimated wealth. All these except the last named have now been published. Probably the final volume giving in detail the results of this census will not be ready much before the close of the year.

BIENNIAL CENSUS OF MANUFACTURES

Of the 57 industry bulletins presenting the results of the 1921 census of manufactures 46 have been printed and the remaining 11 are in type. The complete census will be published in a single octavo volume of about 1,500 pages, the copy for which was sent to the printer on October 23. Proof has not yet been received.

Schedules for the next census of manufactures, which covers the year 1923, have been mailed to all manufacturers concerned with a request that they be filled out and returned promptly. The Bureau is making a supreme effort to collect a large part of the schedules for this census by mail to reduce its cost. Various methods are being used to call attention of manufacturers to the importance of returning schedules promptly. Trade associations and chambers of commerce have coöperated by carrying notices of the census in their trade journals. It has recently been decided also to go one step further and send to Chambers of Commerce lists of delinquents in cities having a population of 10,000 or more (possibly excepting New York City, Chicago, and Philadelphia) asking them to urge manufacturers of their respective cities who are delinquent to make prompt return of schedules.

J. A. H.

EMPLOYMENT STATISTICS FOR CITIES

Social workers and others have urged the necessity for providing in the development of employment statistics for the publication of data by cities. Employment conditions differ widely between cities, and figures for entire states or larger areas are of comparatively little use for indicating changes in trends in particular cities. The United States Employment Service began in 1921 to publish employment figures for 65 different cities in the United States. But publication of these figures was stopped early in 1922 and there was some question as to the accuracy with which the figures portrayed local conditions.

Up to the time these figures were stopped, however, they were the only local employment figures which were being published in the United States except the figures for New York City of the New York Department of Labor, and labor union figures for certain Massachusetts cities. Considerable progress has been made recently in this direction. The following list shows the cities in the United States and Canada for which monthly employment statistics based on employers' payroll reports are now being made available and the agencies publishing the figures.

Cleveland Chamber of Commerce: Cleveland
Detroit Employers' Association: Detroit
Illinois Department of Labor: Chicago, Aurora, Bloomington, Cicero, Danville, Decatur, East St. Louis, Joliet, Moline, Peoria, Quincy, Rockford, Rock Island, Springfield
Massachusetts Department of Labor and Industries: Boston, Brockton, Cambridge, Fall River, Fitchburg, Haverhill, Holyoke, Lawrence, Lowell, Lynn, New Bedford, Peabody, Springfield, Taunton, Worcester
Canadian Bureau of Statistics: Montreal, Toronto, Ottawa, Hamilton, Winnipeg, Vancouver

The Federal Reserve Bank of Philadelphia has recently published monthly employment statistics for 22 cities in the Third Federal Reserve District. The bank has now, in coöperation with the three state departments of labor, extended the collection of employment statistics to cover all of Pennsylvania, New Jersey, and Delaware. A basis for the publication of local figures for industrial localities rather than for separate cities is now being worked out and the bank will shortly begin publication of the data for these districts. They will include the Philadelphia, Pittsburgh, Harrisburg, Newark-Jersey City, and Patterson districts, and the other more significant industrial centers in these states.

The New York Department of Labor is now preparing to extend its analysis of local employment trends and within a few months it will make available separate figures for the following industrial centers: New York City, Albany (including Schenectady and Troy), Utica, Syracuse, Rochester, Buffalo, Binghamton.

R. H.

PROCEEDINGS OF THE EIGHTY-FIFTH ANNUAL MEETING OF THE AMERICAN STATISTICAL ASSOCIATION

WASHINGTON, D. C., DECEMBER 27-29, 1923

PROGRAM

GENERAL TOPIC FOR ALL SESSIONS: *The Statistical Basis for Analyzing the Current Economic Situation, with the Object of Making Forecasts of Business Conditions in General and for the Great Groups of Economic Activities in the United States*

THURSDAY, DECEMBER 27

9.00 A. M.—First Session (Joint Meeting with the American Economic Association, American Farm Economic Association, and American Association for Labor Legislation)

Topic: The Business Outlook

Papers: "The Outlook in the United States"
Charles O. Hardy, University of Iowa
Louis G. Michael, United States Department of Agriculture

Paper: "The Outlook in Europe"
Joseph S. Davis, Food Research Institute, Leland Stanford University

Discussion:
H. C. Campbell, Department of Commerce
R. B. Warren, Federal Reserve Board

12.30 P. M.—Reception at the White House

1.30 P. M.—Joint Luncheon

3.00 P. M.—Second Session, W. M. Steuart, Director of the Census, presiding

Topic: Trade

Paper: "Fluctuations in Retail and Wholesale Trade"
W. Randolph Burgess, New York Federal Reserve Bank

Paper: "The Relation of Mercantile Sales to Other Cyclical Factors"
Willford I. King, National Bureau of Economic Research

Paper: "Forecasting Demand"
Lewis H. Haney, New York University

Discussion

8.00 P. M.—Third Session, Joseph A. Hill, Associate Director of the Census, presiding

President's Address: "Some Fundamental Concepts of Statistics"
Warren M. Persons, Committee on Economic Research, Harvard University

Report: "The Recent Meeting of the International Statistical Institute"
Walter F. Willcox, Cornell University

9.00 P. M.—Smoker and Social Program. A brief business session will precede the smoker

FRIDAY, DECEMBER 28

9.00 A. M.—Annual Business Meeting for Election of Officers, etc.

10.00 A. M.—Fourth Session, Elliot H. Goodwin, U. S. Chamber of Commerce, presiding

Topic: Manufacturing

Paper: "Some Needed Data on Business Cycles"
Francis Walker, Federal Trade Commission

Paper: "Difficulties in Relating Manufacturing Policy to the Business Cycle"
L. H. D. Weld, Swift and Company

Paper: "Automobile Manufacture and the Business Cycle, with a Forecast for 1924"
Raymond B. Prescott, Consulting Statistician

Paper: "The Uses of Employment Statistics"
W. A. Berridge, Brown University

Discussion

2.00 P. M.—Fifth Session, John Hays Hammond, presiding

Topic: Mining

Paper: "Fluctuations in Mineral Output"
George Otis Smith and others, United States Geological Survey

Paper: "The Effect of New Technique on the Past and Future Production of Petroleum"
Joseph E. Pogue, Consulting Engineer

Paper: "The Metals"
E. E. Robie and F. E. Wormser, Assistant Editors of the Engineering and Mining Journal, New York City

Paper: "The Bituminous Coal Industry in Prosperity and Depression"
D. L. Wing, Consulting Statistician, and F. G. Tryon, U. S. Geological Survey

Discussion:

J. B. Umpleby, University of Oklahoma
Allan H. Willett, Bureau of Coal Economics, National Coal Association

8.00 P. M.—Sixth Session, H. C. Wallace, Secretary of Agriculture, presiding

Topic: Agriculture

Paper: "Agricultural Forecasting"
H. C. Taylor, Chief, Bureau of Agricultural Economics

Paper: "Forecasting Corn and Hog Prices"
H. A. Wallace, Editor, Wallace's Farmer

Paper: "Forecasting Agricultural Conditions from Industrial Conditions"
G. F. Warren, Cornell University, and F. A. Pearson, Bureau of Agricultural Economics

Paper: "Forecasting Crop Yields from Weather Conditions"
J. B. Kincer, United States Weather Bureau

Paper: "The Data Necessary for Forecasting the Prices of Farm Land"
R. T. Ely, University of Wisconsin

Discussion:

H. G. Montgomery, Bureau of Agricultural Economics
G. P. Scoville, Cornell University

10.00 P. M.—Smoker

SATURDAY, DECEMBER 29

10.00 A. M.—Seventh Session, W. M. Daniels, Yale University, presiding

Topic: Transportation and Building

Paper: "The Statistical Basis for a Forecast of Railway Traffic with Particular Reference to 1924"
R. H. Aishton, President, American Railway Association

Paper: "Variations in Railway Traffic, Past and Future"
David Friday, Director of the Research Council of the National Transportation Institute

Paper: "Forecasting Building Construction in the United States"
W. C. Clark, S. W. Strauss Company

Paper: "Stabilization of the Building Industry"
E. S. Bradford, Consulting Statistician

Discussion:

M. O. Lorenz, Interstate Commerce Commission
J. H. Parmelee, Bureau of Railway Economics
J. M. Gries, Department of Commerce
Wilson Compton, National Lumber Manufacturing Association
Leo Wolman, New School for Social Research

2.00 P. M.—Eighth Session, A. C. Miller, Federal Reserve Board, presiding

Topic: Banking, Credit and Investment

Papers: "The Use of Statistics in Determining Banking Policies"
W. G. Gephart, First National Bank of St. Louis
L. P. Ayres, Cleveland Trust Company
O. C. Lockhart, National Bank of Commerce, New York

Paper: "The Relation of Monetary Appreciation and Depreciation to the Business Cycle"
Irving Fisher, Yale University

Discussion

MINUTES OF THE ANNUAL BUSINESS MEETING

A brief business session to hear reports was held following the presidential address on Thursday evening, December 27, 1923, President Warren M. Persons presiding.

Reports were submitted as follows:

Report of the Secretary, 1923

The following report is submitted with respect to the condition of the Association:

Membership, December, 1922	986	
Additions during 1923	172	
[1] Deductions during 1923	117	
Net gain during 1923	— 55	
Membership, December, 1923		1,041
Besides members we have:		
Subscribers, mostly libraries	297	
Domestic exchanges	26	
Foreign exchanges	76	
Total members, subscribers and exchanges		1,440

The following deaths have been reported during the year: John Koren (Fellow), Richard C. Lappin.

Four numbers of the JOURNAL and an index, 9 pages, for the volume have been issued during 1923, 529 pages, completing Volume XVIII, which contains two years' publications, 1922 and 1923, a total of 1,089 pages. A total of 19 pages of advertising matter appeared in the four issues at the rate of twenty-five dollars per page.

Three special meetings and dinners of the Association have been held in New York City during the year. The attendance was respectively 85, 310 and 100. One such meeting was held in Chicago and one in Washington. These special meetings should be encouraged wherever the local membership is sufficient in numbers to make them possible, and should be held as frequently as the interest in them will justify.

ROBERT E. CHADDOCK, *Secretary*

[1] Above deductions are as follows:

Deaths reported during 1923	2
Resignations during 1923	35
Dropped because unable to locate	13
Dropped for non-payment of dues	67
	—117

Report of the Treasurer, 1923

Receipts

Membership dues		$5,371.25	
Sales, single copies of JOURNAL and sets		618.55	
Sales, Memorial Volume:			
Sales direct to members	$87.60		
Sales, Macmillan Company	66.30		
Total		153.90	
Advertising, 19 pages at $25		475.00	
Subscriptions to the JOURNAL		1,419.21	
Dividends on B. & A. R. R. stock		148.75	
Balance forward from 1922		352.60	
			$8,539.26

Expenditures

Printing:			
Printing four issues of JOURNAL:			
December, 1922	$1,004.17		
March, 1923	873.81		
June, 1923	940.11		
September, 1923	908.61		
Authors' reprints, December, 1922, to September, 1923, inclusive	212.94		
Handbook issued with September, 1922, issue and unpaid	306.19		
Total printing of JOURNAL	$4,245.83		
Other printing, stationery, etc.	408.18		
Total printing		$4,654.01	
Postage		299.92	
Salaries		1,110.57	
Miscellaneous expenses		72.08	
Balance on hand December 15, 1923		2,402.68	
			$8,539.26

Report on Memorial Volume Fund, 1923

Receipts

Sales to members at $3.50 rate	$87.60	
Sales through Macmillan Company	66.30	
[1] Deficits to date which have been paid out of general funds	471.73	
		$625.63

[1] It should be noted that all debts connected with the publication of this volume have been met in full either from the sales of the volume itself or from the funds of the Association. At present the sales of the volume have not yet met the expense of its publication by the amount of $471.73. However, about 300 volumes still remain in the possession of the Association and each year's sales reduce this deficit.

Expenditures

Deficits for 1920, 1921, 1922	$616.15	
Printing expense	9.48	
		$625.63

Assets

The Association owns without encumbrance seventeen shares of Boston and Albany Railroad stock, besides the unsold copies of the Memorial Volume mentioned above.

ROBERT E. CHADDOCK, *Treasurer*

Report of the Auditing Committee

We have this day audited the accounts of the Treasurer of the American Statistical Association as of the close of business December 15, 1923. We find the same to be correct. All disbursements are properly supported by vouchers and by cancelled checks. The record of receipts and disbursements, as shown by the books of the Treasurer, have been found to be in agreement with the record of bank deposits and the reported cash balance agrees with the bank statement. As is customary, we have made no audit of accounts receivable and payable.

EDWIN W. KOPF,
GEORGE H. VAN BUREN,
Auditing Committee

The reports of the Secretary-Treasurer and of the Auditing Committee were approved and ordered printed in the proceedings.

Committee on Governmental Labor Statistics

(The report of Miss Mary Van Kleeck, Chairman, is presented in full as a separate note in this issue of the JOURNAL on page 76.)

The recommendations of the Committee were approved, the report was accepted and the Committee continued.

Classification of Statistical Workers in Government Service

The following memorandum and resolutions were presented by the Secretary for consideration by the Association:—

By an act approved March 4, 1923, Congress provided "for the classification of civilian positions within the District of Columbia and in the field services."

This act establishes and defines five major divisions of positions which it terms "services," namely, the professional and scientific service, the sub-professional service, the clerical, administrative, and fiscal service, the custodial service, and the clerical-mechanical service.

The act provides, "The professional and scientific service shall include all classes of positions the duties of which are to perform routine, advisory, ad-

ministrative, or research work which is based upon the established principles of a profession or science, and which requires professional, scientific, or technical training equivalent to that represented by graduation from a college or university of recognized standing."

"The clerical, administrative, and fiscal service," it provides, "shall include all classes of positions the duties of which are to perform clerical, administrative, or accounting work or any other work commonly associated with office, business, or fiscal administration."

The act further provides, "Whenever an existing position or a position hereafter created by law shall not fairly and reasonably be allocable to one of the grades of the services described in the compensation schedules the (Classification) board (established for the administration of the act) shall adopt for such position the range of compensation prescribed for a grade, or a class, thereof, comparable therewith as to qualifications and duties."

The Federal Classification Board has not been consistent in allocating the positions of economists, sociologists, and technical statisticians to services, and it has published no class specifications as required under the classification act, yet it can be fairly said that as a general rule it has allocated economists, sociologists, and technical statisticians to the clerical, administrative, and fiscal service, despite the following facts:

1. The work of government economists, sociologists, and technical statisticians is not "to perform clerical, administrative, or accounting work, or any other work commonly associated with office, business, or fiscal administration."

2. The general practice of the United States Civil Service Commission for a number of years has been to require that applicants for the economic, sociological and technical statistical positions shall have had specialized training equivalent to that represented by graduation from a college or university of recognized standing.

3. The proper performance of the economic, sociological, and technical statistical work and the proper administration of the merit system of selection require that the technical positions in these fields in the government service in the future be recruited from persons who have had professional, scientific, or technical training equivalent to that represented by graduation from colleges or universities of recognized standing.

To place the economic, sociological, and technical statistical positions in the clerical, administrative, and fiscal service and not to adopt as the standard for new entrants into the service technical training equivalent to that represented by graduation from colleges or universities of recognized standing, is a distinct backward step which may easily result in filling these positions through political pressure to the detriment of important economic, sociological, and statistical work.

Improper allocations to services has in many instances been accompanied by improper allocations to grades, with the result that salaries for important economic, sociological, and technical statistical positions even as reclassified are too low. Under-allocation is an injustice to the persons now holding the positions. It constitutes a serious impediment to economy and efficiency in the development

of the economic, sociological, and technical statistical work of the government. Low salaries for important positions will not hold the best men and women now filling these positions. Low salaries will not attract to these services the high type of trained employees necessary in growing and developing fields. The low salaries paid by the government for the important positions in economics, sociology, and statistics have been notorious. The action of the Personnel Classification Board has prevented the Classification Act of 1923 from being an effective remedy.

Therefore, be it resolved that it is the sense of the American Statistical Association that technical statistical workers in the Government Service are in fact and should be classified in the professional class.

Further, be it resolved, that the President of the Association appoint a committee of three members not employees of the government of the United States, to take such action as said committee may find appropriate to secure the classification of the technical statisticians in the professional, and scientific services as defined in the Classification Act of 1923 and in the appropriate grades under that service as defined in that Act, that said committee be directed to prepare and file with the Personnel Classification Board, with the chairman of the Committees on Civil Service of the two Houses of Congress, and with the President of the United States, briefs on the proper classification of statistical employees; that said committee be directed to prepare a report regarding what, if any, amendments to the Classification Act it may deem desirable to perfect the administration of said act; and that said committee be hereby authorized to cooperate with any committees of the American Economic Association, the American Sociological Society, or the American Association for Labor Legislation which may be appointed for a like purpose.

The memorandum was approved and after amendment the resolutions as above stated were adopted. The President appointed the following committee as provided in the above resolutions: M. C. Rorty, Chairman, Kate H. Claghorn, Edwin F. Gay.

A final business session was held Friday morning, December 28, 1923, for the election of officers and for other business, President Persons presiding.

It was proposed and adopted that a volume of the JOURNAL, beginning with Volume XIX for the year 1924, shall consist of the four issues of a single year, instead of eight issues, two years' publications, as at present.

Report of Committee on Nominations

The report of the Nominating Committee, Allyn A. Young, Chairman, E. Dana Durand and Wesley C. Mitchell, was presented by the Chairman for final action. The Secretary was instructed to cast one ballot for the nominations as presented. The following officers for the year 1924 were declared elected: President, Louis I. Dublin; Vice-Presidents, R. E. Chaddock, R. H. Coats, M. O. Lorenz; Counsellors, L. P. Ayres, David Friday, W. I. King; Editor, W. F. Ogburn; Librarian, Horace L. Wheeler.

The Nominating Committee reported that they had not been able to select a candidate for the office of Secretary-Treasurer to take the place of R. E. Chad-

dock, who finds it necessary to give up the duties of this office as soon as a successor can be found. On motion of Professor Young the Board of Directors was authorized to fill this vacancy when the proper person can be found. Meanwhile, the present Secretary-Treasurer has agreed to continue as Acting Secretary-Treasurer until the Board is able to make the permanent appointment.

Committee on Fellows

Carroll W. Doten, Chairman, reported for record the agreement of his committee in electing the following members as Fellows of the Association, as provided in Article V of the Constitution: W. L. Crum, Truman L. Kelley, F. R. Macaulay, H. L. Rietz, Carl Snyder.

Joint Advisory Committee on the Census

This Committee was made a permanent standing Committee of the Association at the last Annual Meeting and the Board appointed, according to instructions, W. S. Rossiter for a term of three years, C. W. Doten for a term of two years, and T. S. Adams for a term of one year. A like number were appointed by the Economic Association.

W. S. Rossiter, Chairman of the Joint Committee, reported on the work of the past year. The Committee is recognized by the Director of the Census as an important adjunct to the Bureau. Assistance has been rendered in the planning and publication of the results of the 14th Census, especially in the matter of more scientific presentation of certain topics in the form of monographs, a number of which are either completed or well advanced toward publication. The provision for this permanent Committee is regarded as a significant forward step on the part of the two associations.

The President of the Association announced that the Board has appointed R. E. Chaddock for a period of three years to fill the vacancy on this Committee created by the expiration of the one-year term of T. S. Adams, in accordance with the resolution adopted at the last Annual Meeting which reads as follows: "Resolved, members of said Joint Committee representing this Association shall be appointed by the Board of Directors for terms of one, two, and three years respectively. Members shall be eligible for reappointment. Election of representatives of this Association upon the newly constituted Joint Committee shall take place on or before January 15, 1923, and thereafter the annual vacancy shall be filled at the meeting of the Board of Directors immediately preceding the Annual Meeting of this Association." (Adopted at Chicago, December 29, 1922.)

Committee on Institutional Statistics

The Chairman, Dr. H. M. Pollock, statistician of the State Hospital Commission of New York, presented the following report:

Your Committee on Institutional Statistics takes pleasure in announcing that it has completed the special task of preparing a system of statistical records and reports for the use of institutions for dependent and delinquent children, which was assigned it by your executive committee.

This new statistical system consists of a series of schedule cards for first admissions, readmissions, discharges, deaths and transfers, a set of twenty-three tabular forms and a manual of instructions for filling out the schedules and preparing the annual tables.

The plan adopted by the committee in this work is similar to the one that has been successfully introduced in institutions for mental disease throughout the country.

Negotiations for the introduction of the system in institutions are under way, but no definite results can be reported. It is hoped that the complete manual will be published by the Federal Children's Bureau and thus made available for the use of institutions.

The advantages to be derived from the use of uniform records and reports in institutions of the same kind in the several states and throughout the nation are generally recognized. It seems probable, therefore, that the plan proposed by your committee will be welcomed by many institutions, and it is hoped that it may find favor with state and federal supervisory boards so that its purpose of bringing uniformity and system into a field, where both are lacking, will be fulfilled.

The census of patients and other inmates of institutions taken by the Federal Census Bureau as of December 31, 1922, marks a decided advance in this branch of the Bureau's work. The published results of this census will supply many valuable data concerning the institution population that have not heretofore been available.

The report was accepted and the Committee was continued for another year in order to promote the adoption of the forms and procedures recommended in the full report which the Committee hopes will be published by the U. S. Children's Bureau.

Miscellaneous Business

Through the President a request was made for the coöperation of the American Statistical Association with other scientific societies in the appointment of a Joint Committee of the different associations for the purpose of planning and publishing an Encyclopedia of the Social Sciences. This request was referred to the Board with power.

Dr. Joseph A. Hill brought forward the project of the creation of a Central Statistical Committee or Supervisory Agency to coördinate the statistical activities of the Government at Washington. There was no time for extended discussion and the matter was referred to the Board with power.

Mr. P. P. Gourvich presented a memorandum concerning the professional and scientific needs and opportunities of the Association which was referred to the Board for appropriate consideration.

Resolutions

Resolved, that the American Statistical Association expresses its deep appreciation of the work of the local committee, of which E. A. Goldenweiser is Chairman and W. F. Callander and Dorsey Hyde, Jr., are members, in arranging for this

series of meetings and for the entertainment of the visiting members and their friends.

The Association herewith expresses its thanks to all who have contributed to the success of the meetings by their coöperation in making up the program or in contributing papers on the program, or in consenting to preside over the discussions at the various sessions.

ROBERT E. CHADDOCK, *Acting Secretary-Treasurer*

COMMUNICATIONS

To the Editor of the Journal of the American Statistical Association:

Sir: The publications read before the American Statistical Association annual meetings are of such interest to members of the Association that we have felt that they would be likely to have some interest for the general public if the newspapers in the cities where the meetings are held could receive effective summaries and brief data regarding each meeting. Accordingly, in ample season before the recent meeting of the Association in Washington, the Council delegated one of its members to have charge of the task of securing summaries of the papers and putting them together in proper shape, with sufficient data of each meeting, and having this information duplicated for the use of the Washington papers. This task was carefully performed, though it represented considerable labor both in preparation of material and in getting in close touch with the Washington newspapers. They promised hearty coöperation, but, as a matter of fact, practically none of the material furnished them was used.

Upon inquiry it seems that the Washington daily papers are so surfeited with meetings of various organizations that unless material has a political cast, or something of a somewhat sensational or unusual nature arises, they pay practically no attention to meetings of this kind. Members of the Association who were present in Washington will recall, if they read the Washington papers, that this general policy was clearly marked. Where a Government official presided at a meeting or made an address, the papers make sure to make some reference to it, and also in cases where very striking conclusions were reached which seemed to represent some headline value.

It is probable that no recent meeting of the Association has had papers of greater merit and scientific value than those presented in Washington. They are all well worthy of permanent preservation, but it has been made clear that even with such exceptional material it is and must always be extremely difficult to secure much press attention. This conclusion perhaps does not apply in cases where the Association meets in cities not often favored with scientific gatherings and where in consequence local pride leads them to give considerable space to both papers and deliberations.

Both the officers and members of the American Statistical Association have reason for great encouragement and satisfaction in the very large attendance at the meetings in Washington. It was not many years ago that the meetings of the

Association were attended by a baker's dozen of faithful souls who stood by the organization through thick and thin. Meetings of this kind were profoundly discouraging to the men who had prepared papers with great care and were conscientiously giving of their best. It also placed the Association in a very defenseless position in apportioning time and arranging for coöperation with the other associations meeting at the same place and on the same dates, so that it seemed as though one association, at least, was the dog and the American Statistical Association was the tail, and rather a small and mangy one at that. It is therefore a source of congratulation to the entire membership to realize that our time-honored and important organization is coming into its own, and that where offers of small accommodations for meetings are made on the basis of the old conditions, we are now able to contemptuously reject them and insist upon a large meeting-place.

The meeting in Washington demonstrated that the great number of men who attend these group association meetings and are searching for the most effective and important papers and discussions are now taking the American Statistical Association into account as one of the principal factors in leading them to go to the expense in time and money of attendance at these meetings. The immense increase in the use of figures in the practical affairs of business, politics and finance is steadily making the American Statistical Association one of the most important, and perhaps the most important, organization of the group which meets annually together. It is the one organization which has a definite, distinct mission as apart from theoretical and scientific endeavor.

Very truly yours,

W. S. ROSSITER

MISCELLANEOUS NOTES

JOHN KOREN

In the death of John Koren, the American Statistical Association loses a member and a former president who has devoted much of his time for the past twenty years to its service. Mr. Koren was educated for the ministry, but he was more interested in social welfare than theology and, after a few years in religious work, he entered the field of social investigation to which he devoted the last thirty-five years of his life. He began his statistical work at a time when there were few in the field, and he had the rather unusual distinction of maintaining himself for over thirty years as a professional statistician unaided and unhampered by the duties and responsibilities of an administrative office.

At the annual meeting of the Association in January, 1901, Mr. Koren presented a paper on "Some Statistics of Recidivism in Boston." Apparently this was his first contact with the Association, but from that time until his death, his interest in its work and in promoting its welfare never flagged. The following year he was made a member of the Committee on Publications. Three years later, in 1905, he became chairman of this Committee and, when an editorial board took over the duties of the Committee in 1911, he became the first editor of the "Quarterly Publications." These were critical years for the Association. After the death of President Francis

A. Walker in 1897, the membership decreased during the next ten years from nearly 600 to less than 300. The Publications were often put out as double numbers owing to the lack of material to fill four issues a year. Under Mr. Koren's leadership a new era began. The Publications grew rapidly in size and in the range of statistical work covered.

In 1912 when Mr. Koren became president, the membership had grown to over 600 and the Publications were averaging about 100 pages per issue. Under his leadership as president, the 75th anniversary of the founding of the Association was celebrated in Boston, February 13 and 14, 1914, with fitting ceremonies. This celebration was made the occasion of securing contributions from the leading statisticians of the principal countries of the world on the history and progress of statistical work in those countries up to that time. These papers were later edited by Mr. Koren and published as a Memorial Volume, a work which could probably never have been accomplished, owing to the Great War, if it had not been undertaken when it was, and by a man who had a wide acquaintanceship in the principal countries of Europe. This volume will long stand as a milestone of progress in statistical work and will perhaps serve as the best monument to the memory of him who conceived it and carried it to completion.

CHARLES S. SLOANE

Mr. Charles S. Sloane, Geographer of the United States Census, died on November 3, 1923. He was born in 1859, graduated from Dartmouth College in 1880 with the degree of B.S. in civil engineering. The following year he accepted a position in the United States Surveyor's office in New Mexico, and from that time to the day of his death was continuously employed in the service of the United States Government. His first connection with the Census Bureau dates from 1889. The Bureau as then organized was temporary and when it went out of existence in 1895, Mr. Sloane accepted a position with the Geological Survey. He returned to the Census Bureau upon its reorganization in 1899 and became Geographer of the Census in 1904.

He prepared the Statistical Atlases of the Thirteenth and Fourteenth Censuses, and had charge of the designing and preparation of the illustrations, diagrams, and maps used in Census publications. In connection with the decennial census of population he had the important task of defining and mapping out the enumerators' districts, which, at the Census of 1920, numbered over 90,000. He was Secretary of the United States Geographic Board from 1904 until the day of his death. He occupied a unique position in the organization of the Bureau of the Census, and out of love for his work rendered a most valuable service for a very inadequate compensation.

J. A. H.

New York Meeting of the Association, December 14, 1923.—A special meeting and dinner of the Association was held at the Aldine Club, New York City, on Friday evening, December 14, 1923. More than one hundred members and guests were present. The subject was "What Statisticians Should Know; How and Where Should Their Training Be Given?" Mr. Malcolm C. Rorty presided. The speakers of the evening were Dr. Louis I. Dublin, Statistician of the Metropolitan Life Insurance Company, New York City, who discussed the duties, responsibilities and the educational background of the "executive" group of statistical workers. Dr. Dublin emphasized first, the necessity that the directing statistician should be an expert in his particular line of business,—an able, inspiring leader, who should be fitted to act

as a *liaison* officer between the executive and production divisions of his establishment. He held that with the necessary and inevitable extension of American business undertakings and the realization of their responsibility for public service, there would be more, rather than less opportunity in the future for the statistician who is also of sufficient calibre to act as counselor to the executives of a modern business enterprise.

Mr. Donald R. Belcher discussed the selection and training of staff or supervising statisticians in direct charge of specific inquiries. He drew upon his rich experience in this type of work for the American Telegraph and Telephone Company, and emphasized the primary selection of persons of inherently superior intellectual endowment. He held that the outstanding item in the collegiate preparation of candidates for statistical positions in business was thorough discipline in the physical sciences and in mathematics. This, he said, would tend to develop those characteristics of intellectual honesty, of logical rigor and the capacity for brief, concise, forceful expression so necessary in the work of staff statisticians. He said that the duties of the staff statistician were to have immediate charge of specific inquiries, to collect, tabulate, analyze and interpret statistical materials in such manner that they need receive only editorial treatment at the hands of the executive statistician, whose business it was to apply the results directly to the needs of the executives of a business enterprise. Mr. Belcher's paper is published in full on page (61) of this issue.

Dr. E. E. Lincoln, Chief Statistician of the Western Electric Company, discussed the requisites of personality and training for statistical assistants, the office clerks and field workers; but, like Mr. Belcher, he urged a primary selection of the candidate for one of these more modest positions on the basis of inherent capacity and personal characteristics. He suggested, also, that it would be a service both to candidates and to business establishments if the Association would operate a registration bureau. This bureau, he said, should make reasonable tests of the proficiency of candidates and certify the same to prospective employers. He thought that such a bureau would meet with the cordial support of employers of statistical assistants. He recommended that the Association appoint a committee to consider this registration project. At the present time, candidates for positions in statistical organizations have no means of establishing connection with employers, and employers have no means of gauging the candidate's capacity.

The discussion of the papers was opened by Miss Mary Van Kleeck of the Russell Sage Foundation, who, in addition to other constructive comment, urged the Association to interest itself in the classification of statistical positions under the Civil Service in the Government Departments at Washington. Mr. Lewis Merriam spoke on this subject, also, and outlined some of the specific obstacles which had been met by members of the Association in presenting the claims of statistical employees of the Government to advanced or professional classification. It seemed to be the sense of the meeting that the Association would be of constructive service to statistical science if it took under consideration at the Washington meetings this question of the Civil Service classification of statistical employees. Dr. W. R. Burgess of the Federal Reserve Bank, New York City, outlined some of the problems of selecting and training statistical employees in that organization. He agreed with Dr. Lincoln that a registration service on the part of the Association would be eminently desirable. Mr. S. O. Martin discussed the problems of selecting statistical employees in the field of business research. Professor F. C. Mills of Columbia University urged more cordial coöperation between the statisticians in business service and the teachers of statistics in our colleges and universities. He said that there could be no real ground

for complaint against the teaching of statistics so long as statisticians in business failed to provide suitable teaching material. He urged that statistical offices in American business establishments prepare, as soon as practicable, illustrative practical problems in business statistics. This point was also emphasized by Professor R. E. Chaddock of Columbia University.

Mr. Rorty brought up the subject of preparing a set of standard reference works on statistics for use not only in colleges and universities, but in business offices. He held that this work should be begun as soon as practicable and that some consideration be given the subject at the meetings of the Association in Washington during Christmas week. Dr. Roland P. Falkner expressed his satisfaction at the growth of the Association since he began his pioneer work in teaching statistics, and urged that a concrete educational programme be drawn up as soon as practicable. Dr. Falkner contrasted the present meeting with one which he attended thirty-five years ago and congratulated the Association on its growth and vitality. He expressed his regret that the speakers who had insisted so strongly upon the things that the colleges and universities could not do, had said so little about what they could do. Constructive suggestions would be welcome to all teachers of the subject.

International Conference of Labor Statisticians.—An International Conference of Labor Statisticians was held at Geneva, Switzerland, under the auspices of the International Labor Office, from October 29 to November 3, 1923. The conference was held for the purpose of discussing methods of facilitating the exchange of data regarding labor conditions between countries, with reference more particularly to the classification of industries and occupations, the statistics of wages and hours of labor, and the statistics of industrial accidents. The Conference elected Mr. Armand Julin, Secretary of the Belgian Ministry of Labor and Administration, as president, and Messrs. Coats (Canada), Solinas (Italy), and Platzer (Germany), as vice-presidents. In addition to the very valuable work which the Conference committees accomplished in the way of specific recommendations for gathering labor statistics, it is interesting to note that there was a general recognition of the fact that labor statistics must be considered as part of general statistics, and that international coördination in labor statistics can only be established and maintained in contact with the statistics of population, trade, production, etc.

National Industrial Conference Board Reports.—The National Industrial Conference Board announces the following reports in preparation by its research staff: "The Economic Effects of Minimum Wage Legislation in the United States"; "Analytical Study of Index Numbers of the Cost of Living in the Principal European Countries": "Collective Bargaining in the Metal Trades"; "Hours of Work Legislation in the United States"; "Anti-Trust Policy: A Study of Public Control of Industrial Combination and Business Coöperation"; "Systems of Wage Payment"; "Trend of Railroad Wages and Railroad Performance in the United States"; "Effects of Taxation"; "Population and Food Supply"; "Extent and Incidence of Accidents and Illness in the United States"; "Education and Training for Industrial Leadership"; "Works Councils." In addition, the National Industrial Conference Board publishes each month an index number of the cost of living, of wages, and of employment in the United States.

The Federal Reserve Board will soon issue a special bulletin on prices in the United

States and abroad, in which the price material collected by the Board will be presented for the entire period for which figures have been compiled. The bulletin will also contain comparisons of price statistics in foreign countries and of foreign exchange rates and of prices reduced to a gold basis.

Beginning with January the Board discontinued the publication of a first and a final edition of the Federal Reserve Bulletin, and has consolidated the two into one edition appearing during the first week of each month. The consolidated edition contains all current information previously carried in the final edition, but does not present as many historical tables. The subscription price for the Bulletin has been reduced from $4 to $2.

Dr. John Cummings, formerly Economist and Statistician for the Federal Board for Vocational Education, has joined the staff of the Division of Research and Statistics of the Federal Reserve Board. Two other additions to the division's staff are Mr. W. W. Riefler, who recently returned from service in South America with the Department of Commerce, and Mr. W. R. Stark, previously with the International Banking Corporation in the Orient.

Dr. W. A. Berridge, who was with the Division during the summer months, has constructed an index of factory employment in manufacturing industries based on payroll statistics, and an index of the labor market based on statistics of employment offices. The indices were described in the December and February numbers of the Federal Reserve Bulletin.

Dr. J. Arthur Harris of the Carnegie Institution of Washington has been elected head of the Department of Botany of the University of Minnesota, where he will develop work in Biometry as well as in Plant Physiology.

Dr. Harry S. Will, formerly of Toledo University, is bringing out a series of articles in the December, 1923, and in the January and February, 1924, issues of the *Journal of Educational Research* on "Proposals for Standardizing Measurement in Education." The distinctive characteristic of the theory which Dr. Will builds up is the use of a *standard range* as the basis of his unit. The unit is $\frac{1}{100}$ of the standard range; and to this unit is given the name *kental*. Some interesting simplifications result.

Miss Sylvia L. Parker, Associate in the Department of Biometry and Vital Statistics in the School of Hygiene and Public Health, Johns Hopkins University, has resigned to accept a position in the College of Agriculture of the University of California, where she will be engaged in research. Her work in the new position will begin about May first.

Publication of Proceedings of Annual Meeting.—Special arrangements have been made by the Council of the American Statistical Association for the publication of papers read at the Eighty-fifth Annual Meeting of the Association in book form by the Pollak Foundation. This volume will be available to members of the Association at reduced cost, as noted elsewhere in this issue

Department of Commerce's Publications on the World's Export Trade.—The 67th Congress authorized the Department of Commerce to make a study of the world's trade in agricultural products, and the results of this study will soon be published in a series of bulletins which will be available in limited numbers for distribution. These bulletins will deal with questions of the flow of trade of the important farm products

between all the principal countries, a comparison of present trade with that before the war, particular discussions of how export trade in these products is actually carried out, etc. The first bulletin to be issued will deal with "The Distribution of the Agricultural Exports of the United States," which will be followed shortly by two bulletins on "Methods of Merchandizing American Wheat in the Export Trade."

In organizing this work Secretary Hoover appointed a committee consisting of farmers, representatives of farm organizations, the trade and the U. S. Department of Agriculture to assist in outlining the work and to draw constructive conclusions from the results. The present series of bulletins represent the reports prepared by the Departmental staff and will form the basis from which the committee will draw such conclusions as it sees fit.

Walter Thompson Prize Award.—On February 5, 1924, the Board of Judges on the J. Walter Thompson Company prize essay competition on the subject: "A Statistical Index of the Purchasing Power of Consumers in the United States," announced their decision as follows:

First prize—$1500.00—to Essay entitled "An Index of the Incomes of Factory Workers," by Professor William O. Berridge, Brown University, Providence, R. I., with the assistance of the staff of the "Brown Bureau of Business Research."

Second prize—$800.00—to Essay entitled "Contributions from Budget Studies to the Construction of a Statistical Index of the Purchasing Power of Consumers in the United States," by Miss Emma A. Winslow, West View Park, Riverside, Conn.

Third prize—$500.00—to Essay entitled "A Statistical Index of the Purchasing Power of Consumers in the United States" (recommending The State Employment Office Index), by Richard A. Flinn, Chief, Division of Employment, New York State Department of Labor, 124 East 28th Street, New York City.

The judges of the contest were: Robert E. Chaddock, chairman, Professor of Statistics at Columbia University; Henry Sturgis Dennison, President of the Dennison Manufacturing Company; A. Lincoln Filene, Treasurer and General Manager of Wm. Filene's Sons Company; Stanley Resor, President of the J. Walter Thompson Company, and Allyn A. Young, Professor of Economics, Harvard University.

MEMBERS ELECTED SINCE DECEMBER

Anderson, H. E., Case Pomeroy Co., New York City
Armstrong, Florence A., Institute of Economics, Washington, D. C.
Bandit Brijkishore Bhargave, The Bjarat Stores, Gwalior, India
Blackett, O. W., Harvard University, Cambridge, Mass.
Boody, R. Elizabeth, Comm. on Economic Research, Harvard University, Cambridge, Mass.
Carter, Hugh S., Graduate Student at Columbia University
Chapman, John M., School of Business, Columbia University, New York
Creswell, Charles F., Statistician, Assn. of Life Insurance Presidents, New York City
Dakin, Edwin, Associate Editor, Commerce and Finance, New York City
Ellsworth, Donald W., American Tel. & Tel. Co., New York City
Evans, J. H., Washington, D. C.

Fisher, William C., Dept. of Labor and Industry, Harrisburg, Pa.
Graham, Benjamin, Inter-security Arbitrage, 100 Broadway, New York
Green, Howard W., Director of Research and Statistics, Cuyahoga County Public Health Assn., Cleveland, Ohio
Hamilton, T. R., Port of New York Authority, New York City
Hansen, C. Frederick, W. T. Grant Co., New York City
Hawkins, R. C., New York Telephone Co., New York City
Hedden, Walter P., Port of New York Authority, New York City
Hely-Hutchinson, Maurice, Edward B. Smith & Co., New York City
Herskovits, M. J., New School for Social Research, New York City
Higgins, Edward W., White Motor Company, Cleveland, Ohio
Horwitz, H. B., The Joseph & Feiss Company, Cleveland, Ohio
Howd, C. R., Dept. of Economics, Franklin College, Franklin, Ind.
Jamieson, J. Thomas, Bureau of Internal Revenue, Washington, D. C.
Johnston, Mary, Institute of Social and Religious Survey, New York City
Kirstein, Louis E., William Filene's Sons Co., Boston, Mass.
Lane, Mortimer B., Bureau of Census, Washington, D. C.
Lewisohn, Sam A., Vice-President, Miami Copper Co., New York City
Love, John W., The Plain Dealer, Cleveland, Ohio
McClelland, Ross J., McClelland Claggett Co., New York City
McFall, Robert J., Dept. of Agricultural Economics, Mass. Agricultural College, Amherst, Mass.
Maar, Charles, State Insurance Department, Albany, N. Y.
Moir, Jesse A., New York Telephone Co., New York City
Myers, William I., College of Agriculture, Cornell University, Ithaca, N. Y.
Nash, L. R., Stone & Webster, Inc., Boston, Mass.
Parker, Frank, State Dept. of Agriculture, Raleigh, N. C.
Pearson, F. A., College of Agriculture, Cornell University, Ithaca, N. Y.
Purves, C. M., Agricultural Administration Dept., College of Texas, College Station, Texas
Roberts, George B., Federal Reserve Bank, New York City
Scoville, G. P., Dept. Farm Management, Cornell University, Ithaca, N. Y.
Shaulis, L. L., College of William and Mary, Williamsburg, Va.
Sloan, L. H., Standard Statistics Company, New York City
Spencer, Leland, College of Agriculture, Cornell University, Ithaca, N. Y.
Stinger, Harriet E., Office Manager, H. & S. Pogue Co., Cincinnati, Ohio
Surface, Frank M., Dept. of Commerce, Washington, D. C.
Tandy, Elizabeth, Comm. on Dispensary Development, New York City
Towles, John K., Chase National Bank, New York City
Warslow, H. T., Comptroller, National Lead Co., New York City
Weale, G. J., Public Accountant, 18 Tremont St., Boston, Mass.
Wells, P. V., E. I. DuPont de Nemours & Co., Parlin, N. J.
Williams, W. J., Girard Trust Co., Philadelphia, Pa.
Wilson, Elizabeth, Washington, D. C.

REVIEWS

Generating Economic Cycles, by H. L. Moore. New York: The Macmillan Company. 1923. 141 pp.

In this volume Professor Moore makes his latest contribution to his well-known theory of business cycles. The volume is in the main a reproduction of articles which have previously appeared in various economic journals, but, presented as they here are in connected sequence, they gain in cogency. The data upon which the conclusions are based are so comprehensive, the mathematical analysis is so keen and thorough, the logical presentation is so convincing, and the theory as a whole is so well rounded and each step is buttressed by such a wealth of reference to recognized authorities that one's first feeling on closing the book is that the case has been made.

The theory is well known, but before entering upon a discussion it may be briefly outlined. Rainfall in a considerable number of regions for which Professor Moore has obtained data is found to present cyclical variations, the maxima being separated by intervals of 8 years. Starting with this discovery the theory moves both forwards and backwards—forwards to the evolution of economic cycles and backwards to the cosmical forces which are the cause of the cyclical variations in rainfall.

Looking forwards, it is held that variations in rainfall produce corresponding variations in crops, which in turn result in inverse variations in the prices of staple agricultural products, constituting some three-fourths of the total raw material required by industry. From this rhythmic variation in prices spring the phenomena ordinarily associated with economic cycles.

Looking backwards it is shown that once in every 8 years inferior conjunctions of the planet Venus occur at a time when the planet is in, or very nearly in, the plane of the ecliptic. It is pointed out that the synchronizing of the planet's rotational and revolutionary periods would produce meteorological conditions on the planet of a violent character and differing widely from conditions on the earth. A discussion follows on the nature and properties of the anode and cathode rays developed in a Crookes tube. It is noted that such rays are deflected by a magnet and are capable of precipitating moisture. Authorities are quoted in favor of a probable identity between the anode or cathode rays and the corona of the sun and also the streamers seen in the aurora borialis. Finally it is held as not improbable that such rays passing in the vicinity of Venus at the times of the above mentioned conjunctions may be so influenced as to produce meteorological effects in the earth's atmosphere.

Such is in bare outline the Moore theory of economic cycles. The strength of the theory, however, does not reside in the deductive reasoning but in a searching statistical study of data. The Fourier analysis, following methods developed by Professor Schuster and Professor Turner, is applied to data respecting rain-

fall, to crops in the United States, Great Britain and France, and to American and English price index numbers; and in all cases, after eliminating secular trends, an 8-year cycle is developed: these cycles synchronizing with one another and with the above mentioned inferior conjunctions of Venus. Finally corroborative evidence is found in the exhaustive studies in tree-rings made by Professor Douglass,[1] in barometric studies made by H. W. Clough and F. H. Bigelow [2] and published by the United States Weather Bureau.[3]

The tracing of a connection between business cycles and non-economic forces is a fascinating hypothesis. Should such "generating cycles" prove to be the efficient cause, then it may be possible to provide for them and mitigate their effects as is done with other foretellable events in nature. The discovery, if ultimately verified, is of the first magnitude.

Science, however, advances by a process of attack and defence not unlike the evolutionary process of survival. A new theory cannot and should not be accepted as proven until all the weak places have been discovered and adequately protected. And it cannot be denied that the theory under discussion leaves a number of joints in its armor open to attack. In the first place the Fourier analysis is a somewhat treacherous guide. Given a series of crude data, it can be fitted with a sine curve of greater or less amplitude and almost any wave length. This difficulty Professor Moore himself recognizes. He says (page 43):

> It is well known that Fourier's celebrated theorem . . . if carried out to a sufficient number of terms will reproduce almost any type of graph. . . . When the parameters of this equation are determined from statistical data the question arises as to the significance of the several terms in the Fourier series. Do the successive terms in the sine series correspond to real periodicities in nature, or are they merely formal terms the summation of which will give the observed values of the original data?

The test which he adopts is based on the amplitude of the cycle: and when in a periodogram a cycle of given wave length shows a great amplitude, especially if it stands as a high peak widely isolated from other peaks, the evidence that a cycle of such wave length "corresponds with a real periodicity in nature" is strong. In some cases, however, even when the periodogram shows for a cycle of given wave length a considerable amplitude—especially if two such peaks are near together—it may be unsafe to conclude that the cycle of that wave length really persists throughout the entire period covered by the crude data from which it was computed. For example, suppose that points be constructed precisely indicating for 30 years the annual positions of a 10-year cycle and for the next 30 years the annual positions of a 9-year cycle: each of the cycles having an amplitude of 10.

In this case it is known that the "crude data" represented two cycles, one of 9 and one of 10 years; and, even if it were not known, it is probable that when the "y's" had been scaled and their terminals connected, it would be perceived that

[1] *Climatic Cycles and Tree Growth*, A. E. Douglass.

[2] "An Approximate Seven-Year Period in Terrestrial Weather, with Solar Correlation," *Monthly Weather Review*, October, 1920.

[3] *Report of the Chief of the United States Weather Bureau, 1900, 1901.* Vol. II, "Report on the Barometry of the United States, Canada, and the West Indies."

the graph represented two cycles. But in general the crude data do not lie precisely on the line of the fitted curve. They oscillate above and below it and hence, as such problems arise in practice, the existence of two successive curves would escape observation. Normally, with the above crude data before him the seeker for cycles would construct a periodogram. He would find large amplitudes for single cycles of either 9 or 10 years, and, applying Fourier's formula, would obtain a fairly good fit. The amplitude of the 9-year cycle would be 7.5 and that of the 10-year cycle 7.85 (Figure 1).

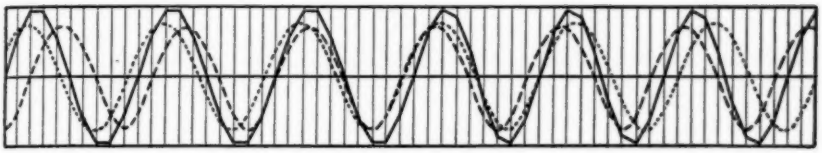

FIGURE I

"Crude data" of a 10-year cycle for 30 years followed by a 9-year cycle for 30 years fitted by Fourier's formula (a) with a 10-year cycle throughout the whole period of 60 years; (b) with a 9-year cycle throughout the whole period of 60 years.

Equation of 10-year cycle $y = 7.85 \sin\left(27°17' + \frac{2\pi}{10}n\right)$

Equation of 9-year cycle $y = 7.5 \sin\left(271° + \frac{2\pi}{9}n\right)$

He might suspect that between them would lie a cycle of even greater amplitude, and, experimenting, would find that a cycle of 9½ years showed an amplitude of 9.57. This cycle on being plotted makes so perfect a fit to the crude data that he would be convinced that he had revealed a real periodicity in nature—the cycle of 9½ years period shown in Figure 2.

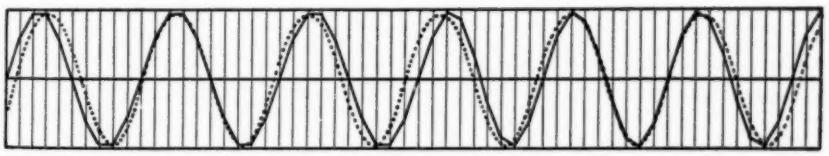

FIGURE II

"Crude data" of a 10-year cycle for 30 years followed by a 9-year cycle for 30 years fitted by Fourier's formula with 9½-year cycle throughout the whole period of 60 years.

Equation of 9½-year cycle $y = 9.51 \sin\left(331°36' + \frac{4\pi}{19}n\right)$

Yet in this particular case we know that he would be in error. The data really represent two cycles of 9 and 10 years respectively.

The above illustration is not fanciful, but has an important bearing on one of the series to which Professor Moore attaches sufficient importance to make it the subject of an entire chapter. Chapter III is devoted to a discussion of Sauerbeck's index numbers. Professor Moore is entirely right in attaching importance to this series. The data are authoritative and cover a long period of time. If an 8-year cycle is a real periodicity in nature, it should be unmistakably revealed in

the 96 years covered by the series—a period long enough to admit of 12 recurrences. It is significant that such a cycle is not revealed even in the analysis made by Professor Moore himself. In the table printed on page 66 the amplitude of the 8-year cycle is shown to be insignificant—$\sqrt{2.12}$ or 1.45. The amplitudes of cycles of 8.7 and 7.4 years are, however, found to be considerable—$\sqrt{14.52}$ and $\sqrt{9.74}$, respectively—and by constructing a composite cycle made up of the two cycles of 8.7 and 7.4 years superimposed upon one another a curve is derived which fits the crude data (after eliminating certain long period trends) with considerable fidelity. But it may be asked, If the 8-year period of approximately coplanar inferior conjunctions of Venus is the efficient cause, why should it result in a composite price cycle of 8.7 and 7.4 years and give no evidence of an 8-year price cycle?[1] Such a composite cycle would result from two periodicities in nature, one of 8.7 years and one 7.4 years not from a single periodicity of 8. In the course of 12 periods an "assumed generating cycle" of 8 years would depart so widely from data representing cycles of either 8.7 years or 7.4 years as to show no fit at all throughout much of the course. Crests of the generating cycle would correspond with almost every possible phase of the cycles represented by the data.

A periodogram (Figure 3) constructed from a portion of the table printed by

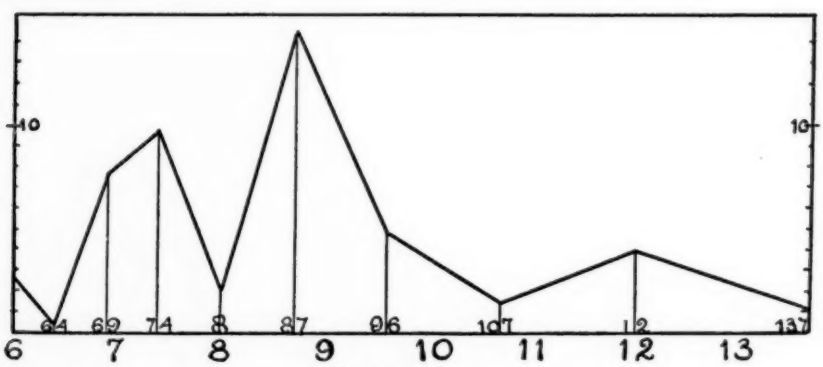

FIGURE III

Periodogram showing the squared amplitude for cycles of the wave length indicated, when applied to Sauerbeck's index numbers of wholesale prices, 1818–1913. Amplitudes as given in "Generating Cycles," page 66.

Professor Moore on page 66 shows high points corresponding with cycles of 7.4 and 8.7 years. There is no evidence of a real periodicity of 8 years, the ordinate at that point being among the smallest shown. The fact that comparatively large amplitudes are shown for cycles 6.9 and 9.6 years suggests that cycles of even greater amplitude than any of those shown may lie between 6.9 and 7.4 or between 8.7 and 9.6. In fact, a cycle of 9 years yields an amplitude of $\sqrt{17.16}$

[1] The periods 8.7 and 7.4 years were obtained (following a method devised by Professor Turner) by computing parameters for cycles whose wave lengths were harmonics of the whole period, *i. e.* cycles whose wave lengths were $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, etc., of 96 years, 8.7 and 7.4 being respectively $\frac{1}{11}$ and $\frac{1}{13}$. By following this method the 9-year cycle whose amplitude is greater than either of those chosen escaped his notice.

—an amplitude greater than that of either of the cycles selected by Professor Moore for his composite cycle of 8.7 and 7.4 years.

In a former review of Professor Moore's work,[1] I applied a 9-year cycle to the crude data (reproduced in Figure 4), after eliminating a long period cycle and secular trend, and obtained a remarkably close fit for the period 1845 to 1900, 55 of the 96 years shown in the table. For the earlier and later years of the table, however, the 9-year cycle may be said to have shown no fit at all. For the later years, however, the data indicated to the eye a cycle of about 7 years as a probable fit. This is suggestive in view of the considerable amplitude shown in the periodogram for a cycle of 6.9 years.

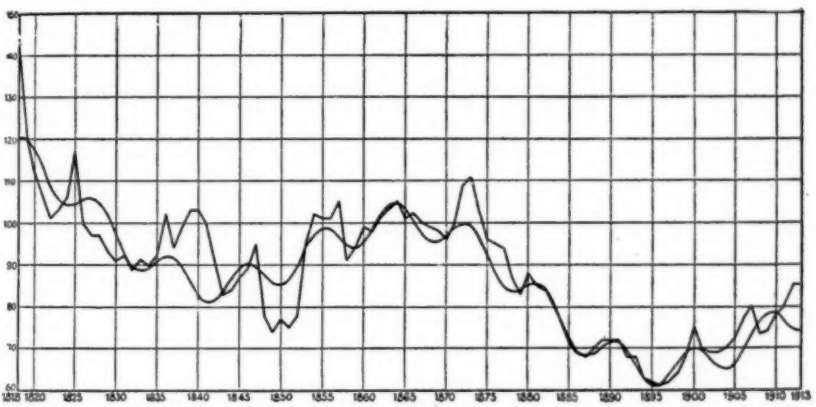

FIGURE IV

Sauerbeck's index numbers of wholesale prices, 1818–1913, fitted with curve $y = 106.5 - 0.3784 + 12.429 \sin\left(122°51' + \frac{2\pi}{54}n\right) + 4.143 \sin\left(55°53' + \frac{2\pi}{9}n\right)$. The fit of the 9-year cycle will be noted.

It may be noted, also, that American price index numbers, 1870–1900, are best fitted by a 9-year cycle. This and other results casting doubt on the 8-year cycle are shown in my review of one of Professor Moore's earlier volumes, "Economic Cycles: Their Law and Cause," in the *Quarterly Journal of Economics*, May, 1915.

As a result of a study of the periodogram and of the hypothetical 9- and 10-year cycles discussed above it is suggested that, if the ups and downs of the Sauerbeck index numbers obey any recurrent law whatever, they may represent a sequence of two or more periodicities of different wave length, rather than a single periodicity of uniform wave length or a composite cycle made up of one wave length superimposed upon another. If the origin of the business cycle is to be sought in human psychology and capitalistic production rather than in cosmic periodicities, it is not unreasonable to suppose that with the progressive speeding up of business the psychological sequence may run its course in a progressively shorter time. This hypothesis is given countenance by recent studies of business cycles. A comprehensive statistical study of business cycles since 1900 (not yet published

[1] *The Quarterly Journal of Economics*, August 1922.

but which the writer has been privileged to see), based on employment, production, trade, prices and other indices of business activity, indicates about three years and six months as the period.

Finally, exception may be taken to the use made by Professor Moore of the studies of Professor Douglass and Professor Clough as verifying his own conclusions. It is to be noted that the latter study is entitled "An Approximate *Seven*-Year Period in Terrestrial Weather, with *Solar* Correlation" (italics mine), and that Professor Douglass, while mentioning a cycle of 7 or 7.3 years (not 8 years, however) does not stress it. What he does stress is the sun-spot cycle of 11.4 years.

On page 85 he says: "We shall find that the sun-spot cycle plays an important part in rainfall."

And in the final summary these words:

> (7) Practically all the groups of trees investigated show the sun-spot cycle or its multiples; the solar cycle becomes more certain and accurate as the area of homogeneous region increases or the time of a tree record extends further back; this suggests the possibility of determining the climatic and vegetational reactions to the solar cycle in different parts of the world.
>
> (8) A most suggestive correlation exists in the dates of maxima and minima found in tree-growth, rainfall, temperature, and solar phenomena. The prevalence of the solar cycle or its multiples, the greater accuracy as area or time are extended, and this correlation in dates, point toward a physical connection between solar activity and terrestrial weather.

A great merit of Professor Moore's work is the publication of the tables upon which his computations are based as appendices to the several chapters. This assists other students who desire to make independent statistical studies. The Moore theory is of such importance that it is to be hoped that such studies will be made and the theory either confuted or confirmed.

PHILIP GREEN WRIGHT

Institute of Economics

Employment, Hours and Earnings in Prosperity and Depression, United States, 1920–1922, by Wilford I. King. National Bureau of Economic Research: New York. 1923. Pp. 148.

The problem attacked by Dr. King in his notable contribution to the study of the effect of the economic cycle on employment and earnings may be stated as follows: What changes in the volume of employment and earnings in the United States occurred between the high point attained in the third quarter of 1920 and the low point reached in the first quarter of 1922? Whether this decline was greater or less than that between similar phases of the cycle in other periods is a question which cannot be discussed at present, but the result of the inquiry is to give for at least one period a quantitative estimate which will serve as a bench mark in future investigations.

Dr. King's investigation makes new departures in the study of fluctuations in employment with respect to scope, method and results. Heretofore, the studies of unemployment regarded as trustworthy have been limited in scope in two

particulars: (1) They have covered only part of the whole field of employment. (2) They have attempted to measure only changes in wages and in number employed. Thus, the valuable monthly reports of the Bureau of Labor Statistics, the most extensive single body of original data in existence, consist of monthly returns of employers in manufacturing industries, and more recently of returns for railroads. No figures are obtained by the Bureau for mining, farming, finance, or trade. Dr. King has essayed the task of estimating the changes in employment in all industries. One conclusion of the utmost importance has emerged. Heretofore, it has been asserted, at least as a possibility, that in times of depression the losses in employment in the restricted field covered by available data were made up to a considerable extent by gains in the uncovered field. Dr. King's figures disprove completely this conjecture. In the face of the fall of employment in all trades and industries, the hypothesis of "shifting" as a considerable form of relief is untenable.

Dr. King's inquiry is also more comprehensive than any other study of employment in that it attempts to measure the decrease in employment resulting from under-employment in the form of short time as well as from separations. The establishment of the small use of part-time employment in this particular period of depression is one of the notable results of the inquiry. Since the publication of Dr. King's book, the Bureau of Labor Statistics has begun the monthly publication of certain data concerning the amount of short time worked; but as yet no attempt has been made to bring this material into any form of index.

The method used by the author in collecting the data was to send to employers questionnaires covering the entire period from the first quarter of 1920 to the first quarter of 1922. Obviously, there is a possibility that a considerable part of the questionnaires were filled in from memory. This possibility is considered at some length in the critical commentary with which the study is prefaced, and an examination of the material has led the author to believe that the estimates from memory are substantially correct. It would have been interesting to know exactly what part of the material was drawn from accounts, since the results of previous attempts to secure information as to unemployment from memory have not been such as to give much confidence in this method. It is possible, of course, that the number of employers returning questionnaires based on records was comparatively so great as to mask the defects involved in the use of questionnaires based on memory. Unfortunately, no exact division of the questionnaires on this basis was possible, since the questionnaire did not ask for the source of the information.

The second step in the method followed by Dr. King was to assume that the sample of establishments for which information was collected was representative of the industry or trade, and to infer that equal fluctuations in employment had occurred in the whole industry or trade. Dr. King admits that for certain of the fields covered—building and construction, public and professional service, etc.—the samples are probably too small to give much validity to the estimates of the total fluctuations in employment. But it is not so much the smallness of the samples which arouses question as the absence of any statement for most of the categories as to the relation of the composition of the samples to the composition

of the wholes. Apparently, the large heads of the census of occupation with three exceptions—manufactures, trade, and transportation—have been assumed to be entirely homogeneous so far as fluctuations in employment are concerned. But this is by no means certain.

An illustration may be drawn from Dr. King's calculation of the fluctuations of employment in the extraction of minerals. From employers in this "industry," 49 reports were received, covering 56,771 persons employed. The total number of persons employed in the extraction of minerals in 1920, according to an estimate based on the census of occupations was 1,120,000. Dr. King assumes that the fluctuations in the number employed by these 49 employers represent the fluctuations in employment in the industry. But the "extraction of minerals" is not a single homogeneous industry. There are included, for instance, anthracite coal mining and bituminous coal mining, industries differing in many important particulars. Were the 56,771 miners making up the sample used for estimating the fluctuations in employment in the "extraction of minerals" group distributed among the various industries composing this category in the same proportions as the 1,120,000 persons employed in the group of industries? Or—to put the matter in another form—were the fluctuations in the employment of the samples of anthracite miners, bituminous miners, etc., calculated separately and then weighted according to the proportion of such miners in the whole group in making up the average of the fluctuations? On these points, no information is given in the text. Obviously, if the component parts of the industry varied considerably in their employment fluctuations, and the sample establishments were drawn disproportionately from any one part, the result may be very far from correct.

Perhaps the most interesting of Dr. King's findings relates to the differences in the extent of fluctuations in employment between large-scale and small-scale enterprises. The small-scale undertakings uniformly show a smaller drop in employment in the period of depression. Dr. King offers a variety of hypothesis to account for this difference, ranging from the possible greater inaccuracy of the questionnaires returned by small employers to the greater reluctance of the small employer to lay off his men. Is it not possible that there is a greater plasticity of wage rates in the smaller establishments, and that reductions in wages and the prices of products, with consequent increase in business, are made more promptly in the smaller establishments? The explanation of the difference in fluctuation on the basis of greater plasticity in wage rates is in accord with a considerable body of economic opinion on the relation of wage rates to the business cycle. Whatever the cause, in establishing the fact of the difference Dr. King has made an important contribution to the study of employment fluctuations.

GEORGE E. BARNETT

The Johns Hopkins University

Predicted Growth of Population of New York and Its Environs, by Raymond Pearl and Lowell J. Reed. New York: Committee on Plan of New York and Its Environs. 1923. 42 pp.

This pamphlet presents the results of an investigation into the growth of population of New York City and the surrounding territory by Professors Pearl and Reed at the instance of the Committee on Plan of New York and Its Environs. The report is descriptive in nature and consists: First, of an account of the mathematical theory which forms the basis for the method employed, and second, of an exposition of the results secured by applying the method to the specific data of the New York region.

Professors Pearl and Reed assume that the recorded numbers of persons living on a given area at various dates in the past afford a satisfactory basis for determining the numbers that will inhabit that area at various dates in the future, and that the problem of forecasting may be reduced to the mathematical-statistical one of fitting a curve of suitable type to the observed data and projecting it into the future. The selection of an appropriate formula for this purpose rests on a consideration of the following five factors:

1. A finite area must be taken in any practical population study.

2. There must be some finite limit to the number of persons that can live upon a finite area.

3. The lower limit to population is zero.

4. History indicates that population growth proceeds in cycles, each superposed upon the last, and each marking the duration of a more or less definite cultural epoch.

5. History also indicates that during each epoch or cycle the rate of growth varies with time. The curve of population rises slowly at first, gradually accelerates until a point of inflection or point of maximum slope is reached, and then decelerates, slowly approaching an upper asymptote or limit.

It is shown that the curve

$$y = \frac{b}{e^{-ax} + c}$$

where y represents the population of a finite area at time x, satisfies all of these postulates except number 4. It has a lower asymptote of zero, a finite upper asymptote, and is of the general shape suggested in postulate 5. No attempt is made to portray the cyclical waves of population growth. It is regarded as preferable to treat each cycle separately. Certain amplifications of this general form of curve are discussed but are not subsequently employed in the application to the specific data of this study. The only departure made in certain cases from the formula given above is to add a constant term to the right-hand member of the equation, thus giving a lower limit other than zero, suggesting the existence and completion of earlier cycles.

The first specific problem was to forecast on the basis of decennial data from 1850 to 1920 compiled from the United States census reports the population of each of three irregular concentric subdivisions of the region surrounding and including New York City, and covering parts of the states of New York, New

Jersey, and Connecticut. The total population of the three areas combined was also forecast independently from data extending back to 1790. A separate prediction was also made for the population of the present area of New York City itself. Many of the results of these projections are too detailed to be given here and only the most important features can be mentioned.

It is found that although the population of none of the areas has yet passed the point of most rapid increment growth, by the year 2000 conditions will in all cases roughly approximate those of the saturation point or asymptote. At that time the population of the total area will have increased from its present value of 9,000,000 to about 29,000,000. This increase of over 200 per cent is unequally distributed among the sub-areas, however, for it is estimated that the intermediate of the three concentric areas will contain more than 6 times its present population, while the corresponding ratios for the innermost and outermost sections are about 2.5 and 3, respectively. New York City, which forms at present about four-fifths of the central sub-area, will also contain about 2.5 times its present population.

Forecasts of the trends of certain elements of the population of New York City are also made by projecting the ratios of each of these elements to the total. No appreciable trends in the percentages of the total population falling in each of the major age groups were found, so that the age distribution of the future population is predicted to remain about as at present.

In the case of the negro and foreign-born populations, pronounced trends in the ratios of these groups to the total were found and the same formula was applied to the ratios as had been used in forecasting the total population. The projected ratios were then used in conjunction with the forecasted population figures for the entire city to determine the actual numbers of these elements to be expected at future times. By the year 2000 the negro population is expected to have tripled and to constitute 5 per cent of the total population as against 2.5 per cent at present. The foreign-born population is expected to increase from its present number of 2,000,000 to approximately 3,750,000, which will be about the peak for this element of the population; thereafter there will be a slow decline in the number of foreign-born, gradually approaching 1,500,000 as an asymptote. This peculiar movement is caused by the decline in the percentage of foreign-born which has been in progress since 1860 and is expected to continue.

Perhaps the primary function of a review of a purely descriptive work such as the one under consideration involves only a summary of the descriptive material. It is the privilege of reviewers, however, to make a critical inquiry into the precepts and hypotheses implicitly underlying the application of methods to the specific problem. In the case of the present study the reviewer cannot feel convinced of the value of the results for the reason that the given data do not seem susceptible of a mathematical-statistical treatment.

The population of an isolated area which can be neither augmented by immigration nor diminished by emigration may change only through a net excess of births or deaths. The only area which is strictly thus isolated is the entire habitable surface of the earth. Population changes in such an area are de-

pendent largely upon conditions of fecundity, resistance to disease, and availability of food supply. Since these elements tend to change but slowly, the population here might be accurately represented by a mathematical function, with, however, the reservation that even in this simple case fortuitous events in the nature of wars, plagues, and famines would tend to effect a permanent displacement of the curve with respect to time.

In an area that is not thus isolated the actual changes in population are the resultant of the natural changes in conjunction with the aleatory increments or decrements due to migrations. Since there is no logical basis for assuming that the economic and social factors which determine these migrations are functionally related to time, and since, further, history actually indicates that these factors are variable in intensity and erratic in duration, the application of a time-function to any series of which they form a part seems untenable, for purposes either of description or of projection.

It may be maintained, however, that a method which cannot be thoroughly defended by logical means may still in application be found to yield results of sufficient practical precision. But such assertion requires demonstration. In the present case of projecting the population of the New York region, more confidence in the forecasts would be inspired if it were shown that the application of the method to data in the more remote past resulted in satisfactory predictions for the more recent past. In this connection it is noted from examination of the trend of the percentage deviations of the observed values from the graduated values for the entire New York region during the first half of the period studied, that a projection of these data through the second half of the period would probably have resulted in very great discrepancies.

It is particularly difficult to believe that accurate conclusions as to the upper limit of the population of a given area can be deduced from the population history, or even that the population necessarily tends to approach a limiting value in the manner implied by the curve. The very industrial and business expansion which tends to attract increasing numbers of persons to a certain area may in time proceed to such a degree that it actually expels population from the area.

When attempt is made to forecast the trends of component elements of the population such as the negro and foreign-born constituents, the complexity of the problem is greatly increased by the preponderance of the fortuitous increments. The negro population of New York City from 1910 to 1920 was largely augmented by an influx of negroes from other sections of the country. And any accruals to the foreign-born population must necessarily result from immigration; indeed, in order to maintain even a constant number of foreign-born, there must be sufficient immigration to replace the losses due to deaths and departures. To what extent these factors will influence the magnitude of future populations it is obviously beyond the domain of the statistician to predict.

HAROLD M. FLINN

New York City

An Elementary Treatise on Frequency Curves and Their Application in the Analysis of Death Curves and Life Tables, by Arne Fisher. Translated from the Danish by E. A. Vigfusson, with an introduction by Raymond Pearl. American Edition, Macmillan Company: New York. 1922.

Mr. Fisher's most recent work is an extension of a preliminary note submitted to the Casualty Actuarial Society of America several years ago. Later a more detailed discussion was published in the *Scandinavian Insurance Magazine*. Now, at the earnest solicitation of friends here and abroad, he presents an exposition of his method of constructing mortality tables from the tabulated facts for deaths according to cause and age. The so-called "exposure to risk" plays no part in the method. So far, the method seems to be applicable only to the data for male lives. Mr. Fisher emphatically states that the procedure is designed to supplement, not supersede, the conventional and time-tried plans of vital statisticians and actuaries, such as are outlined in standard works—the Text-book of the Institute of Actuaries, the texts of Newsholme and the Actuarial Society of America. A vast amount of material now lies fallow because exposure-to-risk data cannot be obtained to complement the supply of facts on causes of death by sex and age. This applies particularly to the nosologic experience for insured occupational groups and for other life insurance experiences. Mr. Fisher tells us that he has endeavored to substitute the inductive principle for the pure empiricism underlying the method of Halley and all who succeeded him in vital statistical work.

The essence of Fisher's method follows:

1. *The d_x curve of the life table for a cohort of, say, 1,000,000 males entering observation at age 10, is considered to be generated as a compound curve of a limited number (say 8 or less) of component curves of either the LaPlacean-Charlier or Poisson-Charlier type.*

In order that the reader may understand frequency curve analysis and nomenclature as developed by Charlier and his compatriots, Mr. Fisher presents a digest of his larger work, "Mathematical Theory of Probabilities," in the first 104 pages of his present essay. The lack of this helpful introductory material was a stumbling block to American statisticians and actuaries who tried conscientiously to understand his preliminary note in the Casualty Actuarial Society Proceedings. Patient, faithful examination of this prefatory material is recommended to students who may wish to understand Mr. Fisher's method of mortality table construction.

2. *The method of induction requires the discovery of two separate groups of statistical constants or parameters of the component or subsidiary curves.*

(a) Statistical characteristics, or semi-invariants, which determine the frequency distribution of proportions at attained ages to "all causes of death" of certain groupings of specified causes, diseases or conditions. The characteristics used in these determinations are: the (1) mean, (2) dispersion, (3) skewness and (4) excess (for each subsidiary grouped-cause component).

(b) The areas of each subsidiary or component curve.

Mr. Fisher requires a working hypothesis as follows:

> The relative frequency distribution of deaths by attained ages, classified according to a limited number of groups (generally 8 or less) of causes of death among the survivors of the original cohort of entrants, tend to cluster around certain ages in such a way that it is possible, from biological considerations, to estimate in practice with a sufficiently close degree of approximation the statistical characteristics or semi-invariants of the relative frequency distributions of the component curves, corresponding to a previously chosen classification of causes of death (into 8 or less subsidiary groups).

Most of Mr. Fisher's critics have stumbled all over this hypothesis. His translator, Mr. E. A. Vigfusson, points out that these critics did not see that the analysis is based upon two attributes of human mortality: (1) the purely *quantitative* attribute of attained age at death and (2) the purely *biological*, or qualitative, attribute of causes of death.

The closing section of the working hypothesis implies that Mr. Fisher has been able, from biological considerations, to select *a priori* the statistical characteristics of a group, or of groups, of causes of death required in 2-a (above)! The method rests upon the working hypothesis, and its rationality depends upon the validity of an *a priori* choice of certain parameters. Mr. Fisher and his critics both agree that this validity has not been established, but must be proved or disproved either by independent methods and principles, . . . *or through additional biological studies*. The author frankly submits that he has advanced a temporary, preliminary and unproved supposition. Something is supposed to be true, and there is an attempt to ascertain whether certain facts fit together better than they do with any other supposition hitherto tried. The facts fit together well enough, as exemplified by the remarkably close agreement of Fisher's d_x values with those computed by conventional methods by Professor Glover for approximately the same area (Fisher: *U. S. Registration Area, 1910*, and Glover: *Original U. S. Registration States, 1909–1911*).

In applying his method to Michigan data for the years 1909–1915, the parameters chosen "on biological grounds" to meet the requirements of 2-a (above), for nine groups of causes of death are given in the table on the following page.

This table of parameters is prefaced by the working hypothesis: "The frequency distribution of deaths according to age of the above groups of causes of death among the survivors of an original cohort of 1,000,000 entrants at age 10 can be represented by a system of frequency curves determined by the following characteristic parameters." Mr. Fisher then says: "From these parameters and from well-known tables of the probability or normal frequency curve and its various derivatives, it is easy to determine the frequency distribution for any desired interval." Assuming acquaintance with the first chapter of the book, the calculation of the desired frequency distributions certainly is easy, but where did Mr. Fisher get the parameters? On page 161 another and different table of parameters appears for use in building a table for Massachusetts males, 1914–1916. While the Michigan parameters were also "previously used with success for certain Scandinavian data," these Massachusetts parameters are said by Mr. Fisher to have been used by him in

PARAMETERS*

Group	Causes characteristic of	Mean (Years)	Dispersion (Years)	Skewness	Excess
I.....	(Extreme old age)	79.5	9.5730	+.1056	+.0546
II.....	(Middle old age)	70.5	12.8000	+.0967	+.0126
III.....	(Early old age)	65.5	13.6870	+.1248	+.0650
IV.....	(Middle life)	59.5	17.0890	+.1790	−.0106
V.....	(Early middle life)	55.5	19.9411	+.0555	−.0367
VI.....	(Pulmonary tuberculosis, etc.)	44.5	16.0352	−.0124	−.0272
VII-b.....	(Early life occupational hazard)	257.5	12.1552	+.0008	−.0005
VIII-a.....	(Middle life occupational hazard)	Poisson-Charlier curve: Modulus 28.5 years; eccentricity 1.0001.			
VIII-a.....	(Childhood)	Poisson-Charlier curve: Modulus 13.5 years.			

* For diseases and conditions included in groupings, see pp. 133–138 of Fisher's book.

the construction of a mortality table for England and Wales, 1911–1912. What were the biological principles underlying the calculation of these necessary preliminary parameters? How, from principles alone, without empirical data, did Mr. Fisher calculate those parameters correctly to three or four decimal places?

On page 183, Mr. Fisher recapitulates the processes employed in computing his several tables. Until Mr. Fisher tells us how and why he made his *a priori* choice of "a system of frequency curves based upon the hypothesis that the distribution of deaths according to age from typical causes of death can be made to conform to those postulated frequency curves whose parameters are *known or chosen beforehand*," it would be futile to go on with a description of the remaining seven or eight processes which lead finally to d_x values.

The subscriber wishes to acknowledge the assistance of Professor Charles C. Grove in the preparation of this review.

E. W. Kopf

Applied Business Finance, by Edmond E. Lincoln. Chicago: A. W. Shaw Company. 1923. Pp. xviii, 772.

This book, originally published in the summer of 1922 and reissued in a second edition early in 1923, provides a very useful discussion of the principles and problems of business finance. It is a "practical" book, and should be of as great service to the man of affairs as to the student. Indeed, the approach is largely that of the man of affairs. "Throughout," the author asserts, "the point of view aimed at is that of the thoughtful, conservative, and thoroughly successful business man;" and primary emphasis is placed upon "those underlying principles of finance which have stood the test of the best practice and which have been heartily endorsed by leaders in the business world." Concrete illustrations of both good and bad financial practice are numerous, and they are largely drawn from business experience sufficiently recent to be vividly comprehended by the reader. Many forms illustrative of various aspects of

financial transactions are reproduced, and the course of the argument is frequently supported by valuable statistical data. The entire treatment is realistic, with regard to "intangible" psychological factors as well as objective facts.

As compared with most books on business finance, Dr. Lincoln's study possesses a number of more or less distinctive characteristics. It recognizes the influence of the business cycle upon the problems of finance, particularly as it has made itself felt during the war and post-war period. It deals with the average-sized business concern, unincorporated as well as incorporated, rather than with very large corporate undertakings. It is concerned primarily with the financial problems of manufacturing and trading enterprises, rather than with those of railroads and public utilities. It emphasizes the problems involved in raising working capital, instead of devoting itself disproportionately to the financing of fixed assets. It analyzes the tasks of internal financial management, aside from the matter of working capital, by giving extended attention to the financial aspects of business processes which are commonly regarded as falling outside the domain of business finance.

A cursory survey of the chapter outline of the book will serve the twofold purpose of indicating concretely the content of the study and the nature of the foregoing characteristics. As a starting-point for the analysis, the first two chapters are devoted to a discussion of the relationship of the business cycle to business finance, and of the importance and normal problems of the average industrial concern. By way of dealing with the promotion period, the next two chapters discuss the launching of the enterprise and the form of its organization. Two chapters are then devoted to the instruments of long-time finance. Three chapters deal with matters of financial plan: raising long-time capital without security issues, raising long-time capital by means of security issues, and raising new permanent capital for established concerns. Three chapters, dealing, respectively, with the business and the investment banker, business finance and the stock market, and customer ownership and employee ownership, take up the problems of marketing or distributing securities or other evidences of business proprietorship. By way of digression, two chapters are then devoted to the relationship between accounting and business finance, dealing with financial statements and their interpretation, and comparative statements of condition and financial standards. Four chapters discuss the problems of raising working capital—dealing, in turn, with the business and the commercial bank, the business and the commercial paper house, trade acceptances and bank acceptances, and miscellaneous methods of raising working capital. Then follow five chapters, covering almost two hundred pages, which are seldom found in books on business finance, and which are primarily concerned with matters of general business administration. These chapters deal, respectively, with purchasing in its financial aspects, financial aspects of producing goods, the financial results of size, financial aspects of selling goods, and business finance and mercantile credit. The two concluding chapters are devoted to a discussion of the distribution of earnings and financial difficulties and failures.

The emphasis placed in this book upon the ordinary problems of moderate-sized manufacturing and trading concerns constitutes a valuable departure from the usual procedure; for the traditional treatise is generally confined to corporation finance, draws its chief impulse from the practices and vicissitudes of the large corporation, and finds much of its illustrative material in the field of railroads and public utilities. The attention devoted to the relationship between the course of the business cycle and the problems of finance reflects the growing recognition of the importance of cyclical fluctuations in the adjustment of business enterprise, and indicates, in very suggestive fashion, how the results of recent research in this field may be applied specifically in the financing process. The discussion of working capital is both more comprehensive and more adequate than that usually found in the texts on business finance. Aside from these merits, however, it appears that the scope of the volume is so broad as to impinge unduly, and without profit, on the general field of business administration. Here are some of the concrete questions which the author puts to himself for detailed discussion (pp. 42–43):

> How shall purchases be financed, and when and how shall they be made? How extensively should a concern purchase? What does it cost to produce goods? What factors most influence the cost of production? How can waste be eliminated? How are costs and earnings affected by the size of the producing unit? When shall a concern sell and when refrain from selling? What are the chief problems of commercial credit and what should be the relation of the sales department to the credit department? What policy should be followed regarding terms of sale, collections, discounts, and the like? What is the proper price policy? When does it pay to advertise?

It is submitted that most of these questions are questions of finance only in the sense that every business problem, under a money economy in which the profit motive is the dominant spur to economic enterprise, is a financial problem. The author's attempt to provide information "as to how a business might be conducted in such a way as to meet its obligations and put sound dollars back of all securities issued" has carried him far afield. The chapters devoted to a discussion of what must be deemed essentially business problems might well have given place to a more detailed consideration of the instruments of finance, the formulation of financial plans, the administration of income, and the problems of failure and reorganization.

While the book is written in readable style, it frequently lapses into journalistic phraseology and homiletic mood.

I. L. Sharfman

University of Michigan

The Mortality Experience of Industrial Policyholders, 1916–1920—A Contribution to the Public Health Movement in America by the Colonial Life Insurance Company of America, John Hancock Mutual Life Insurance Company, Life Insurance Company of Virginia, Metropolitan Life Insurance Company, Prudential Life Insurance Company of America. New York; 1923. 150 pp.

This volume is the work of thoroughly competent statisticians familiar both with the methods of collection, tabulation, and analysis of vital statistics and with the interpretation of the results in terms of significant public health achievements. The preparation of the material, gathered from five important industrial insurance companies, was in charge of a committee, formed in 1914, of which Dr. Louis I. Dublin was chairman. This committee laid its plans for a comprehensive analysis of the statistics, organized in some cases the statistical work practically from the ground up, suggested in other cases such changes in the handling of the statistics as would secure uniform treatment of the material in the several companies, adopted the standard methods of classifying causes of death followed by the Federal Census Bureau, adopted a system of querying vague and indefinite causes of death and agreed upon uniform methods of computing from the weekly returns of premiums paid the average number of persons exposed to the risk of death. The graphic and textual material was prepared by Mr. E. W. Kopf and Mr. George H. Van Buren. Throughout the discussion the sources of error in the basic data are kept constantly in mind, and the report is an excellent critical evaluation of statistics of causes of death. For this reason as well as others the report will prove invaluable to all students of vital statistics.

This volume is of special interest because it treats of the mortality experience of the wage-earning classes, a group of the population which has been neglected in most statistics because of the difficulty of securing the basic information. Most life insurance statistics relate to the middle class and to the well-to-do, while national statistics include the entire population without regard to classes. With the notable exception of a similar study of the *Mortality Experience of Industrial Wage-Earners and Their Families, 1911–1916*, published by the Metropolitan Life Insurance Company, comprehensive data relating to the working classes of the population have been lacking. This volume covers a much larger industrial experience and carries up to 1920 the analysis of mortality by cause and age for the insured wage-earning population.

Space is lacking to review all the important findings. The total number of deaths included in the survey was over 1,300,000 while the total number of years of exposure was nearly 115,000,000. Each important cause of death is discussed separately and analyzed by age, the mortality in the wage-earning classes is compared with that in the general population in the United States death registration area, and the trend of the rates is shown for the five-year period. The most important cause of death among the wage-earning population as among the general population was tuberculosis; the rate was higher but declined more rapidly in the former than in the latter group during the five-year period studied. The discussion of the mortality from influenza emphasizes the enormous losses caused by the epidemic in a single group of the population.

Owing to the failure of one or two of the companies to furnish the necessary basic information unfortunately no analysis of mortality by sex could be made. This is the more to be regretted since peculiar elements of selection which appear to operate in the choice of insured women in ordinary insurance business probably did not affect this industrial insurance experience. The omission of information as to sex seriously limits the conclusions, for example, relating to puerperal mortality, though so far as the age and sex constitution of the insured group did not change during the five-year period the conclusions relating to the increase in death rates from causes connected with pregnancy and childbirth would, of course, remain unaffected. In view of the marked influence which the rule that in case of death from a puerperal cause complicated with influenza the former was the preferred cause seems to have had in increasing the mortality ascribed to pregnancy and childbirth, especially in 1918, it would be of interest to know how much of an increase, if any, there would have been if in such cases influenza had been the preferred cause—as was the rule followed, for example, in the statistics for England and Wales. In this connection, also, it would be of interest to have a table showing the age constitution of the insured group in each of the five years of the period.

Another question of considerable importance to which more discussion might have been devoted is the more exact definition or description of the group of industrial insurance policyholders to which the present statistics applied. So far as possible the study was limited to white policyholders; but no analysis was attempted, for example, by nativity, state or place (urban or rural) of residence, or occupation. The statement is made, however, that "it is quite possible that three out of every four men, women, and children above one year of age in the wage-working families in the cities of the United States and Canada are included in this survey," a statement which, if it suggests on the one hand the importance of an analysis of the population group at risk, on the other hand throws into bright illumination the comprehensive character of the materials with which the report deals.

Great credit is to be given to the Life Insurance Companies for their recognition of the value of such a coöperative study and to the committee in charge of the work for their grasp of the importance in public health work of having an adequate statistical analysis of results achieved.

ROBERT M. WOODBURY

The Federal Reserve System, by Henry Parker Willis. New York: The Ronald Press Company. 1923. Pp. 1765.

After nine years of operation the Federal Reserve System has recently entered upon a new phase in its development. Having come into existence simultaneously with the outbreak of the war the System was during the first years of its life profoundly influenced by the banking and monetary conditions arising out of the struggle. After the United States joined the Allies the System's energies were devoted chiefly to assisting the Government in supplying its fiscal needs; while

in the years immediately following the armistice its operations were affected largely by the violent economic readjustment in this country and in the world at large. It is only since 1921 that the System has had an opportunity of functioning under fairly stable domestic conditions, though still in a world deeply affected by the economic upheaval caused by war. The System is now doing pioneer work in finding new methods of banking administration as well as in the development of new guides to credit policy. This is, therefore, a particularly opportune time for the appearance of a study of the more recent operation of the Federal Reserve System in the light of the conditions which preceded its establishment, the principles upon which it is founded, and the early history of its activity. Consequently one turns to Mr. Willis' study of the development of the Federal Reserve System with the expectation of finding help toward a fuller understanding of the problems with which the Federal Reserve System is confronted.

Of these problems the most fundamental is to what extent the Federal Reserve System can influence business trends and moderate through the exercise of credit control the extreme fluctuations in business and credit. What are the means of control as developed by actual experience? Broadly speaking, they are the exercise of discretion in the granting of loans, discount rate policy and open-market operations. Mr. Willis touches upon the question of discretion as a means of credit control when he describes the first steps in the establishment of standards for paper offered for discount and the difficulties encountered in this respect; but he does not present either a historical account or a current description of the relationship between the loaning activities of the reserve banks and the member banks, or of the position of the Federal Reserve Board in relation to these activities.

To the problem of discount rates the author devotes a considerable number of pages. He discusses the initial problems arising out of the determination of the level at which reserve bank rates should be established in relation to market rates for money. In a later chapter he relates the story of the elimination of preferential rates on paper secured by Government obligations, the raising of the rates in November, 1919, the application of progressive rates by some banks (a plan of which the author does not approve) and of inter-district borrowing. The author's conclusion is that "the resumption of the control of discount rates (in 1919 and 1920) must be regarded as having been theoretical, due to the fact that the raising of rates was invariably deferred until such time as the movement of prices had far outgrown such control." It is not clear what is meant by "invariably," since the text refers to only one series of rate advances, those of 1919 and 1920. The problem is taken up again in connection with a description of the "inflation period." Here the author states that the System was created for the purpose of the conservation and proper apportionment of credit, but had not prior to 1920 been able to exercise this function. He expresses his opposition to the rationing of credit, and proposes instead a strict definition of eligibility of paper which would automatically direct credit to legitimate ends. There is some question, however, to what extent eligibility for rediscount is now a proper standard in view of the enormous volume of outstanding Government obligations

which have practically removed all limits upon the creation of eligible paper. In a subsequent place the statement is made that in 1920 the raising of discount rates was effective in slowing down the rate at which credit expansion was proceeding; yet a little later a topic heading reads "Discount Rates Unimportant," and the author voices the opinion that the reserve system had not been able to exercise proper credit control at the proper stages of business developments for the reason that it had failed to avail itself of its power to enter the open market and thus to become the determining factor in the credit situation. Just what rôle in credit control the author assigns to discount rates and to what extent he believes in their actual or potential effectiveness as moderators of fluctuations in business conditions is not clearly apparent.

The author's views upon open market operations are expressed only incidentally, though there is no doubt of his belief that the reserve banks, like the central banks of Europe, should be active traders in the market and, by being in close touch with it, should become the decisive factor in the credit situation. The reserve bank's failure to do this is construed by Mr. Willis as an abandonment of the most important function of the banks and as resulting in their reduction to the status of mere emergency institutions rather than of full-fledged agencies of credit administration. The author does not explain, however, the character of open-market operations which the banks should undertake, nor the relation of these operations to the volume of reserve bank credit in use. There is no mention of the change which has taken place in the System's open-market policy from one aimed chiefly at fostering the acceptance market to one forming an integral part of credit policy, nor of the relationship between the growth in the volume of security purchases in 1922 and the decline in discounts, followed by developments in the opposite direction in 1923, while the total volume of earning assets, *i. e.*, of Federal reserve credit in use, remained fairly stable throughout the period. And yet the extent of this compensatory tendency as it affects the System as a whole and the individual reserve banks has been in the foreground of reserve banking discussion during the past two years.

It is in the first and second parts of the volume, devoted to the legislative vicissitudes of the Reserve Act and to a description of the System's organization, that the author makes his most valuable contribution to the history of banking in the United States. As assistant to the Indianapolis Monetary Commission in 1897, as expert of the House banking committee, which fathered the Federal Reserve Act, as Chairman of the Technical Committee on organization of the Federal Reserve System, as Secretary of the Federal Reserve Board from 1914 to 1918, and as Director of its research work until sometime in 1922, Mr. Willis had unique qualifications for the preparation of this historical account, as well as access to material not available to other writers, including the files of Mr. Carter Glass, who was Chairman of the Banking and Currency Committee of the House in 1913 and contributes an introduction to this volume. The book contains in a general appendix all the principal drafts of the Reserve Act and all the amendments made in the law after its enactment; in addition chapter appendices reproduce the principal documents relating to the period and subject matter discussed in the text. This material adds greatly to the value of the volume for

reference purposes and explains its bulk, so that the contents of the book are not as formidable as the mere number of pages would lead one to believe.

In conclusion, it should be stated that the volume is in substance a record of the author's connection with the reserve bank idea extending over more than a quarter of a century prior to, during, and after the passage of the Federal Reserve Act, and that this may explain the personal reactions which at times take the place of objective interpretation in the text. Much of the book produces the impression of having been written at different times in the past and incorporated into the present volume without much modification on the basis of subsequent developments. In fact, the author tends to measure the system's achievements not so much in terms of its actual performance as with reference to the standards and functions which were in the minds of the framers of the Act a decade ago.

E. A. GOLDENWEISER

Washington, D. C.

Restoration of the World's Currencies, by R. A. Lehfeldt, D. Sc. London: P. S. King & Son. 1923. 146 pp.

This little book is one of the best and sanest on the important subject of which it treats. The author is Professor of Economics in the University of the Witwatersrand, Johannesburg, South Africa, and the author of "Gold, Prices and the Witwatersrand," published four years ago. He writes from the standpoint of an expert on the gold situation, having been for many years a close observer of South Africa, the largest gold producer in the world and "exceptionally dependent on this single industry, more so even than Chile on nitrate or Australia on wool."

The title of the book is not fully indicative of its scope, for its purpose is to further not simply the restoration but the stabilization of the world's currencies and to do so not simply in relation to gold but in relation to commodities.

In his first chapter Professor Lehfeldt addresses himself to the principles of a just adjustment. He points out that the British owners of government bonds have a just interest in a restoration of the buying power of the sovereign but in varying degrees according to the date at which they lent their money to the Government. The buying power of the sovereign fell in the ratio of 100 to 43 between 1914 and 1919, the average buying power (weighted according to the value of the government debt incurred) being 51.4 per cent of pre-war buying power (page 22). A somewhat similar calculation made by me (*Stabilizing the Dollar*, page 157) shows that our Liberty loans were floated at an average price level of 195 per cent of the pre-war price level, indicating an average buying power of 51.2 per cent of pre-war buying power. Professor Lehfeldt gives more emphasis to the interests of bondholders than to that of wage earners (since wages can be more promptly adjusted), differing in this respect from Professor Cassel. On the other hand, he does not make justice the only controlling consideration. He concludes, Chapter I, with the words:

> But justice, even if it could be effected more thoroughly than is in fact possible, will not alone provide a solution. We have to consider the methods by which stabilization of currency is possible, and the practicability of carrying them out.

These methods he classifies under two heads, (1) managed paper currencies and (2) gold standard methods, the former of which, although theoretically feasible, he rejects as too easily subject to political manipulation and as not effecting stable foreign exchange.

But, like all real students of the subject he brushes aside the common contention, or illusion, that gold itself is very stable. "The drop in the value of gold due to the war has had the effect of making many mines unpayable and reducing the annual output by one-third."

Nor is the author unmindful of the fact that the "value of gold," like that of paper money, is subject to the influence of governmental policies.

He gives some space to the consideration of the compensated gold dollar, commonly associated with my name. Of this he says: "This plan is not only ingenious, but has serious practical merits to recommend it. . . . There can be no doubt that the system would work and that it would confer great benefits on a world plagued with ill-regulated currencies; it would, however, be necessary to convince the commercial and political classes of its merits. Mere conservatism should not be put forward as an argument, but it is permissible to claim that the simple gold standard, being familiar to every one, should be adopted as the means of reforming the present deplorable situation, and that if a system involving a principle new to the business world is to be introduced at all, its advocacy might be deferred in order not to delay the present remedy, the need for which is urgent."

While I can see no reason to believe that deferring the advocacy of the compensated gold dollar would help forward any other plan of stabilization (the opposite seems more likely) I do agree that if the author's plan, or any other workable plan, should become acceptable to legislators in the immediate future it ought to be adopted, even though it be a makeshift, pending whatever period of education may be necessary "to convince the commercial and political classes" of the merits of the compensated gold dollar.

But it remains to be seen whether legislators will even today look with less favor on the compensated dollar plan, which would be relatively inexpensive, than on the author's plan (substantially the same as that proposed by B. M. Anderson, Jr., eleven years ago), a plan to valorize gold, however costly the process.

Professor Lehfeldt proposes that the United States and Great Britain, the two chief gold-producing nations (contributing over four-fifths of the world's production) should enter into an agreement to officially control the production of gold in order to control its value, somewhat as Brazil valorized coffee and, still more, as the diamond producers regulate the diamond market. In periods when prices tended to rise the International Commission charged with this function would close some of the least productive mines, and prevent the development of new ones, paying compensation in each case. In periods when prices tended to fall the Commission would stimulate gold production.

Besides thus regulating the supply of gold, Professor Lehfeldt would also consider the regulation of demand, thus regulating banking and currency.

He recognizes the difficulties involved and the possibility that limits might be

reached which would make impossible further adjustment in one direction or the other.

The author proceeds to suggest definite procedures for different classes of countries, according to whether devaluation or resumption is to be expected as well as other practical considerations. The German reparation problem is not overlooked. He also revives the suggestion of making the sovereign, the dollar, etc., easy multiples or submultiples of each other.

Not the least valuable part of Professor Lehfeldt's book is that in which he discusses the proper standard of deferred payments. His practical and acute discussion of this old problem should be refreshing to all who have followed the older academic essays on the same subject. His main conclusion is that a composite wholesale commodity standard is about as nearly ideal as we are likely to get in a long time.

In a word, Professor Lehfeldt's book is distinctly of the sort which contributes to the progress of world thought and policy.

IRVING FISHER

Yale University

Construction of Mortality Tables from the Records of Insured Lives, by Ray D. Murphy and Percy C. Papps. Actuarial Studies, No. 2.

This little book of seven chapters and 72 pages contains a great deal of information for the student who desires to know how mortality tables are constructed from the records of insured lives. Indeed, it goes back of the stage of construction and describes in much detail the sources and methods of recording the data.

Tables may be constructed from the death rates among the lives insured, from the rates of termination by death of policies in force, from the rates of termination by death of amounts insured. In recording the exposures and deaths in an experience, the actuary may choose any one of three methods, the policy year, the calendar year, or the life year. The authors outline the procedure in each case and give numerous valuable suggestions. They very wisely caution the reader, however, that many of these suggestions may not be applicable to the investigation in question. Peculiarities may exist in the practice of the company which require individual study and treatment.

In chapter four the method of handling the data is described and has to do chiefly with the tabulating, punching, sorting, and listing by means of the Hollerith and Powers machines. In the next chapter, full details of the formulas are given, which express the exposed to risk at age $x+1$ in terms of the exposed to risk, existing, withdrawals, deaths, and other functions at age x. A chapter is then devoted to construction of tables with more than one decremental factor, such as termination by withdrawal, marriage, re-marriage, and so forth, and numerous formulas and references are given. Some interesting observations on the basic difference between death rates and rates of discontinuance due to the different forces acting to produce them appear in this chapter.

The book closes with a miscellaneous chapter containing an account of the

Final Series method, King's method, and the compilation of select tables where data have not been traced on policy year basis.

The literary style is good, the tabular forms and explanations are clear, and on the whole the book is a successful presentation of methods for the actuarial student studying for examinations. The actuary who really is about to construct tables based of insured lives, however, will undoubtedly examine the work of Hardy, Meech, and others, as well as many papers on this subject in actuarial literature before venturing on so important a task.

JAMES W. GLOVER

University of Michigan

The Twelve-Hour Shift in Industry, by the Committee on Work Periods in Continuous Industry of the Federated American Engineering Societies. New York: E. P. Dutton & Company. 1922.

In April, 1911, soon after the results of the Pittsburgh Survey began to be made public, Mr. Charles M. Cabot of Boston, one of the principal minority stockholders in the United States Steel Corporation, moved at the annual meeting of the stockholders, that a committee be appointed to investigate the truth of the statements contained in an article entitled "Old Age at Forty," in which conditions of life and work at the mills were pictured in such a manner as to challenge the attention of the industry. A year later, this committee reported. Among other findings of their investigations they stated, " . . . we are of the opinion that a twelve-hour day of labor, followed continuously by any group of men for any considerable number of years means a decreasing of the efficiency and lessening of the vigor and virility of such men."[1] The twelve-hour day remained, however, in a large number of the steel mills. When Mr. Cabot died in 1917, he left a trust fund to be used for the purpose of investigating social and industrial conditions to the end that industrial hardships might be made known and remedied. This fund has been used mainly for carrying on his work to improve labor conditions in the steel industry. Among the uses to which the Cabot fund has been put were grants in 1920 for an investigation of the progress made in the steel industry in changing from the two-shift day and, later, of the technique of changing from the two-shift system to the three-shift system in the steel industry.[2] These studies were followed in 1921 by a grant to the Committee on Work Periods in Continuous Industry of the Federated American Engineering Societies for a further study of the shift problem in continuous process industries. This report was published in December, 1922. In August, 1923, the American Iron and Steel Institute announced the proposed installation of three shifts to take the place of two in the steel industry and, so far as can at present be foretold, the long day in the steel industry has gone for good.

How much of the responsibility for this change is due to the pressure of public opinion in favor of a shorter work day; how much of it is due to the fact that

[1] United States Steel Corporation. *Report of Committee of Stockholders*, April 15, 1912, p. 6.

[2] Horace B. Drury, "The Three-Shift System in the Steel Industry," in *Bulletin of the Taylor Society*, February, 1921; *ibid.*, "The Technique of Changing from the Two-Shift to the Three-Shift System in the Steel Industry," a report to the Cabot Fund, May, 1922 (Proof Sheets: not Released for Publication).

industrial conditions were favorable; and how much of it to the fact that a large body of carefully organized data was available to indicate the technique of changing from a two-shift to a three-shift schedule, cannot, of course, be accurately measured. Probably all three of these, and other elements entered into the situation.

The Federated Engineering Societies' report on the twelve-hour shift in industry is particularly significant in that it brought together in a way never before attempted the experience of industry with shift work and pointed out on the basis of this experience, the means by which the shorter day could be established. The committee having the investigation in charge was made up of eight engineers; the actual work was done by an economist and an engineer. They themselves may have had definite convictions about the twelve-hour day versus the eight-hour day, but the report is a bare statement of facts and opinions of others, and of conclusions based on the evidence assembled. No theory is exploited; no *a priori* arguments are justified. Picturesque writing and human interest stories are gratefully lacking. So far as the evidence permitted, the complete picture is presented even though qualifications necessary for the truth sometimes weakened certain statements. And out of it all stands the conclusion that in this country "there are upwards of forty continuous-industries operating more or less completely on a shift system. They employ between 500,000 and 1,000,000 wage-earners on shift work. . . . There are 300,000 wage-earners working on twelve-hour shifts." This is, of course, a small proportion of all industrial workers since, "relatively, only a small part of industrial work, 5 to 10 per cent, is on processes which require continuous operation and the number of workers is relatively few." The investigators found that while discussion of the long day had previously centered around the steel industry, it was only because the extent of the practice elsewhere was unknown and "there are several times as much shift-work outside of the steel industry as there is in it, and approximately as many men working twelve hours." "Because", however, "of its size and the large proportion of continuous work, and also because it has been one of the slower of the continuous-industries to move towards the abandonment of the twelve-hour shift, it is almost as important a factor in the field of this study as all the other continuous industries put together. The metal industries, other than steel, are at the present time almost without exception on a three-shift basis."

The report is divided into three parts. The first is a general summary of the detailed reports. In Part II, Horace B. Drury presents the results of his investigation of "the extent of two-shift work in continuous-process industries other than the manufacture of iron and steel" and "the experience of those manufacturers who had changed from two-shift operation to the three-shift or some other system." Part III is devoted to Bradley Stoughton's study of "the technical aspects of changing from a two-shift to a three-shift system in the iron and steel industry."

The method used in the investigation consisted of interviews with plant executives and others who could provide the required information, observation and interpretation of the evidence thus assembled. Exact data for the most

part were not available. Mr. Drury reports: "It was only with much difficulty that even the names of the continuous-industries could be ascertained. It has been rare to find any one person, whether government official, leader of industry, labor leader, or general student of industrial problems who could give the names of more than a few such industries. In the great majority of cases it was not believed by members of these groups that there was in existence such a thing as a twelve-hour day, save for the reports which had been heard concerning it in the steel industry. Even in the case of the trade associations in the continuous-industries, and of individuals who have had wide and lifelong knowledge of these industries, as a rule no one is informed correctly regarding shift practice. Hardly a person can speak with assurance or accuracy regarding the practice in the various sections of the country in the matter of two-shift or three-shift operation."

In addition to surveying the extent of continuous work in American industry, the Drury investigation set out to answer certain other questions, related to such problems as the alternatives to the twelve-hour shift, technical difficulties of changing from two-shift to three-shift operation, and other factors to be considered in making this change; also the results of such a change on the number of shift workers, quantity and quality of production, absenteeism, labor turnover and industrial accidents, wage rates and the use made by employees of their extra hours of leisure. The general opinion of managers regarding the two-shift as against the three-shift system was canvassed and the extent to which plants once having changed from two shifts to three returned to the two-shift system.

Some of these questions are much more adequately treated than others, largely, probably, because of the amount of information available. Cost figures, for example, are confidential to the individual corporations and are not available for outside inspection. Hence quantitative measurements of many factors are impossible. This circumstance necessarily weakens somewhat the conclusions of the report, which are based on interviews and opinions. Nor does Mr. Drury claim that he has covered all of the field in his investigations or that he has done more than secure samples of experiences. How these samples were chosen is not told, but a variety of opinions was secured from a variety of industries. Not all plants that changed from long to shorter hours reported satisfactory results; some have returned to the earlier schedules; not all plants secured increased or better output, in not every plant were the employees better satisfied and in many of them factors other than the shorter day *per se* were influential in the results achieved. "But the weight of the evidence and the most positive statements are in favor of the three-shift operation."

Mr. Stoughton's problem was to determine how the change could be made from the twelve-hour shift in the steel industry. His method was similar to Mr. Drury's. From the experience and conclusions drawn therefrom of plants in the steel industry and other occupations which had made the change, a set of conditions, technical and otherwise, necessary for a successful adoption of a shorter day in the steel industry was formulated. It is interesting that some of these the public heard of as prevailing when the big change was made in the steel industry in 1923, and some of the results predicted have actually followed in that industry.

There are some questions these reports do not answer. For example, there is no discussion of the difference in the effect of shorter hours on different industries or as related to different kinds of work such as those purely repetitive and machine controlled, as contrasted with those requiring judgment and discretion. Nor is there any conclusion regarding comparative effects of going from relatively long to relatively shorter hours as contrasted with the effect of shortening only an hour or two a day. Still, too much should not be expected in a pioneer work and students of industrial conditions as well as industrial managers should thank the authors of this report for digging up and pointing out for further, more detailed study the shift industries and prevailing customs regarding hours of work. To the extent that this is the first attempt to measure the situation at all, it marks a long advance in our knowledge of one phase of a subject about which there have been many opinions but little well-considered evidence.

MARGARET LOOMIS STECKER

New York

Financial and Operating Ratios in Management, by James H. Bliss. New York: The Ronald Press Company. 1923. Pp. xii, 396.

From the author of this book who is Comptroller of Libby, McNeill & Libby, and who has long been interested in administrative accounting, we should expect a thoroughly worth-while and constructive piece of work. To quote from the Preface:

> The principal aim of this book is to develop certain standard ratios for the use of managing executives in securing more effective control of the finances and operations of their business.
>
> * * * * *
>
> While the book is primarily intended for business executives and accountants, it aims also to be helpful to anyone having or expecting to have a financial interest in a company—bankers, credit men, individual investors, etc.

The chapters which will probably be of most general interest and importance are the following:

Chapter	I,	Introductory.
"	IV,	Accounting Statistics as an Aid to Effective Financial Management.
"	V,	Use of Financial Statistics by Business Men, Bankers, Investors, and Accountants.
"	VI,	Measures of Business Efficiency—Financial and Operating Ratios.
"	XXII,	Manner in Which Capital Is Invested in a Business.
"	XXIII,	Sources from Which Capital Is Secured.
"	XXV,	Analyzing Financial Statements.
"	XXVI,	Reading Changes in Financial Position from Reports.
"	XXVII,	Preparation of Financial Statements.
"	XXVIII,	Preparation of Statistics for Business Executives.

Practically every financial ratio of any significance derived from income statements and balance sheets has been worked out for groups of representative companies in the more important industries. Many interesting averages have been computed.

The author is broad-minded in his treatment and recognizes clearly that accounting and statistics are only a means to an end—better administrative control. However, his analysis is perhaps somewhat lacking in economic background. Also, the general presentation and discussion is almost never enlivened by any concrete illustrations which would show the significance of some of these financial relationships in practice. It would have been distinctly useful had the author pointed out more definitely his conclusions as to the comparative value of the several ratios in management. Perhaps, also, it might have been helpful to give less attention to *averages* and more attention to "safe limits."

For several years the reviewer of this book made an intensive study of all operating ratios and financial standards in practically every important line of industry, covering a period as far back as usable records can be secured. He completed this study with the feeling that, while certain clearly defined limits and standards can be fixed for the average concern within a given industry, yet surprisingly few businesses are in all respects close to the *average*. A few show much better than an average performance, while a surprisingly large number show a condition either slightly under the average or much below it.

The personal factor in management and the peculiar conditions surrounding a given company, even in the competitive field, frequently cause many of its balance sheet ratios to diverge markedly from any estimated "normal," even though the financial performance, as measured by net return on capital invested, is perfectly satisfactory.

However, the author himself would probably admit most of these limitations on the study, and, as already stated, Mr. Bliss is to be commended for producing this book, because it should help to make business men think more seriously about the desirability and practicability of establishing certain safe standards to guide them in their financial operations. Facts such as are here presented should be of assistance in the movement "to put more science into business."

EDMOND E. LINCOLN

RECENT LITERATURE

STATISTICAL BOOKS

DUMAS, S. and K. O. VEDEL-PETERSEN. *Losses of life caused by the war;* edited by H. Westergaard. (Publications of the Carnegie Endowment for International Peace. Division of Economics and History.) Oxford Press, 1923. 191 pp.

EAST, EDWARD M. *Mankind at the crossroads.* New York: Scribner's, 1923. viii, 360 pp.

FISHER, ARNE. *Elementary treatise on frequency curves and their application in the analysis of death curves and life tables;* tr. from the Danish by E. A. Vigfusson; with an introduction by Raymond Pearl. New York: Macmillan, 1923. xv, 240 pp.

HARDY, CHARLES O. *Risk and risk-bearing.* Chicago: University of Chicago Press, 1923. xix, 400 pp.

KARSTEN, KARL G. *Charts and graphs, an introduction to graphic methods in the control and analysis of statistics.* New York: Prentice-Hall, Inc., 1923. xl, 724 pp.

NATIONAL INDUSTRIAL CONFERENCE BOARD. *Changes in the cost of living, July, 1914–July, 1923.* New York, 1923. viii, 36 pp.

—— —— *Experience with mutual benefit associations in the United States.* New York, 1923. viii, 155 pp.

—— —— *Tax burdens and exemptions.* New York, 1923. viii, 159 pp.

—— —— *Wages, hours and employment in American manufacturing industries,* July, 1914–July, 1923. New York, 1923. vii, 154 pp.

PEARL, RAYMOND. *Introduction to medical biometry and vital statistics.* Philadelphia: Saunders, 1923. 379 pp.

SCHLUTER, W. C. *The pre-war business cycle, 1907 to 1914.* New York: Longmans, 1923. 189 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 243.)

SCHOTT, SIGMUND. *Statistik.* 3te Aufl Leipzig: B. G. Teubner, 1923. 128 pp.

WRIGHT, HAROLD. *Population.* New York: Harcourt, Brace and Co., 1923. xii, 180 pp.

GOVERNMENT PUBLICATIONS

UNITED STATES.

—— Bureau of the Census.

Annual Report of the Director of the Census to the Secretary of Commerce for the Fiscal Year Ended June, 1923. 1923. 46 pp.

No. 152. Mortality Statistics, 1921. 1923. 112 pp.

Abstract of the Fourteenth Census of the United States, 1920. 1923. 1303 pp.

Mortality Rates, 1910–1920. 1923. 681 pp.

—— Bureau of Education.

No. 40. Report of a Survey of the State Institutions of Higher Learning in Kansas. 1923. 160 pp.

No. 49. Statistics of State Universities and State Colleges for the Year Ended June 30, 1922. 1923. 16 pp.

—— Public Health Reports.

No. 40. "Cancer Mortality Rate in a Group of Insured Persons." 1923. 48 pp.

No. 44. "The Notifiable Diseases in States, 1922." 1923. 154 pp.

No. 45. "Analysis of Cancer Mortality in a Group of Insured Persons." "Reports of the Health Section, League of Nations." 1923. 41 pp.

No. 48. "The Collection of Morbidity Data." 1923. 59 pp.

No. 49. "Summary of Vital Statistics, England and Wales, 1922." 1923. 50 pp.

No. 50. "Death Rates in Registration Area, 1922." 1923. 76 pp.

MICHIGAN DEPARTMENT OF HEALTH. Public Health, December 1923. "Medical Service in Rural Districts." 21 pp.

COMMONWEALTH OF MASSACHUSETTS: Division of Vital Statistics.

Annual Report of the Vital Statistics of Massachusetts, Births, Marriages, Divorces and Deaths, for Year Ending December 31, 1922. 202 pp.

STATE OF NEW YORK: STATE HOSPITAL COMMISSION.

Annual Statistical Review of the Insane in the State Hospitals and Private Licensed Institutions. For Year Ending June 30, 1920. 1921. 368 pp. For Year Ending June 30, 1921. 1922. 194 pp.

STATE OF WASHINGTON.

—— Department of Efficiency.

Financial History of the State of Washington for the Period, Statehood to March 31, 1921. 432 pp.

General Fund Tax Roll History for the Period Statehood to December 31, 1922. 73 pp.

Fourth Biennial Report of the Joint Board of Higher Curricula to the Governor of Washington, Part One, 1922–1923. 1923. 50 pp.

ARTICLES IN PERIODICALS

GIORNALE DI MATEMATICA FINANZIARIA. 1923. Vol. V. No. 4. C. Goldziher: "Sulla determinazione delle constant della formola." C. E. Bonferroni: "Sulla formola del montante." A. Dell' Agnola: "Sul Calcolo del tasso di una rendita immediate."

GIORNALE DEGLI ECONOMISTI.

October, 1923. E. Corbino: "I porti da ancona a venezia." G. Majorana: "Le scienze economiche e statistiche al congressa di Catania."

November, 1923. R. Dalla Volta: "Sulla sviluppo delle dottrine economiche." M. Boldrini: "Una lezione sugli strumenti da calcolo aritmetico."

December, 1923. P. Gorini: "Mercati della cirenaica."

INTERNATIONAL LABOR REVIEW.

October, 1923. "The Work of the Ministry of Social Policy in the Kingdom of the Serbs, Croats and Slovenes." V. Hyvonen: "Coöperation in Finland." "Some Forms of Inspection in Agriculture."

November, 1923. K. Krzeczkowski: "Social Insurance and International Legislation." D. Pap: "Labour Legislation in Hungary." M. Nedelkovitch. "The Labour Market in the Kingdom of the Serbs, Croats and Slovenes."

December, 1923. "The Fifth Session of the International Labour Conference." F. Hawelka: Workers' Sickness Insurance in Austria." "The Eight-Hour Day in Germany in 1921 and 1922."

BULLETIN MENSUEL DEL'OFFICE PERMANENT, INSTITUT INTERNATIONAL DE STATISTIQUE.

November, 1923. "Statistique des prix de gros." "Novembre-indices des prix de gros (avec graphique)." "Novembre-indices des prix de detail et du cout de la vie." "Production Minerale." "Cours du change." "Banques de circulation." "Chomage."

THE JOURNAL OF ACCOUNTANCY.

November, 1923. W. F. Weiss. "Arbitration." L. G. Peloubet: "Balance-Sheet Uniformity." J. A. Cull: "A Tax on Living Expenses."

December, 1923. P. W. Pogson: "The Practical Application of the Theory of Accounting for Supplies."

January, 1923. F. Vierling: "The Interest Question." C. H. Brook: "Branch Accounting."

THE JOURNAL OF THE INSTITUTE OF ACTUARIES.

November, 1923. W. P. Elderton and H. J. P. Oakley. "Mortality of Annuitants, 1900–1920. A Report on Graduation and Presentation of Monetary Tables." E. H. Lever: "Further Notes on the Mortality Experienced by Life Tenants under Revisions."

JOURNAL OF THE INSTITUTE OF BANKERS.

November, 1923. W. C. Smith: "Balance Sheets of Joint Stock Companies and Traders from the Point of View of the Banker as a Lender."

December, 1923, and January, 1924. H. Higgs: "British Government Finance," in two parts.

LE MUSÉE SOCIAL. REVUE MENUELLE.

October, 1923. O. Hesnard: "Les Partis politiques en Allemagne, avec avant-propos de M. Henri Lichtenberger."

November, 1923. Et. M. Saint-Leon: "Les deux C. G. T."

December, 1923. G. M. Crivelli et P. Louvet: "L'Australie et le Pacifique."

MONTHLY LABOR REVIEW.

September, 1923. "Purchasing power of the dollar, 1913 to 1923, as computed by the U. S. Bureau of Labor Statistics." E. Frankel: "How Germany settles industrial disputes."

October, 1923. M. T. Waggaman: "Extension of 'family wage' system in France and Belgium."

November, 1923. C. Hookstadt: "Estimated annual number and cost of industrial accidents in the United States." E. Boehler: "Present economic situation of the German student body."

December, 1923. R. S. Billups: "Bread making in the modern bakery." E. V. Yohe: "Protection of workers under Mexican State labor laws."

JOURNAL OF THE ROYAL STATISTICAL SOCIETY.

March, 1923. R. Duffield: "Discussion of the Registration of Disease." John Hilton: "Statistics of Employment Derived from the Working of the Unemployment Insurance Acts." "Wholesale Prices of Commodities in 1922." "New Zealand Study in Seasonal Fluctuation of External Migration."

May, 1923. A. W. Flux: "International Statistical Comparisons." E. C. Snow: "Trade Forecasting and Prices." H. H. Wolfenden: "On the Methods of Comparing the Mortalities of Two or More Communities." H. G. Stead: "The Correction of Correlation Coefficients." J. Y. Hart: "Notes on the Rate of Sickness of Elementary School Teachers."

July, 1923. D. R. Wilson: "On Some Recent Contributions to the Study of Fatigue." A. L. Bowley: "Death Rates, Density, Population and Housing."

J. C. Dunlop: "Misstatement of Age in the Returns of the Census of Scotland." F. Y. Edgeworth: "Mr. Correa Walsh on the Calculation of Index Numbers."

THE ECONOMIC JOURNAL.

September, 1923. J. R. Bellerby: "The Controlling Factor in Trade Cycles." E. G. Mears: "Financial Aspects of American Immigration." F. Y. Edgeworth: "Index Numbers According to Mr. Walsh."

THE QUARTERLY JOURNAL OF ECONOMICS.

November, 1923. A. Berglund: "The United States Steel Corporation." F. D. Graham: "The Theory of International Values Reëxamined."

METRON.

I–IX–1923. F. P. Cantelli: Sulla oscillazione delle frequenze intorno alla probabilità." L. I. Dublin: "The possibility of extending human life." F. Savorgnan: "Nuzialita e fecondità delle case sovrane d'Europa." E. von Fellner: "Die Verteilung des Volksvermögens und Volkseinkommens der Länder der Ungarischen Heiligen Krone zwischen dem heutigen Ungarn und den Successions Staaten."

BIOMETRIKA.

August, 1923. Seimatsu Narumi: "On the General Forms of Bivarite Frequency Distributions Which Are Mathematically Possible When Regression and Variation Are Subjected to Limiting Conditions." E. S. Pearson: "Natural Selection and the Age and Area Theory of Dr. J. C. Willis." A. A. Mumford and M. Young: "The Interrelationships of the Physical Measurements and the Vital Capacity." Oskar Anderson: "Uber ein Neues Verfahren bei Anwendung der 'Variate Difference' Methode."

OTHER BOOKS

ABELL, SIR WESTCOTT. *The ship and her work.* London: Gee and Co., 1923. 114 pp. (London School of Economics and Political Science. Studies in Commerce. 2.)

ATHEARN, WALTER S. and others. *The religious education of Protestants in an American commonwealth.* New York: Doran, 1923. 580 pp. (The Indiana Survey of Religious Education. v. 1.)

BARNES, HARRY. *Housing, the facts and the future.* London: Benn, 1923. 450 pp.

CAMPBELL, P. C. *Chinese coolie emigration to countries within the British Empire.* London: King, 1923. xxiii, 240 pp. (London School of Economics. Studies No. 72.)

COLLINS, Ross W. *Catholicism and the Second French Republic,* 1848–1852. New York: Longmans, 1923. 360 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 250.)

FISK, G. M. and P. S. PEIRCE. *International commercial policies with special reference to the United States.* A textbook. New York: Macmillan, 1923. xii, 322 pp.

LEES-SMITH, H. B. *Second chambers in theory and practice.* London: Allen and Unwin, 1923. 256 pp. (London School of Economics. Studies No. 71.)

NAHOUM, JULES. *The key to national prosperity. A presentation of foreign trade in its connection with the development of national prosperity.* New York: Dutton, 1923. xiii, 381 pp.

OGATA, KIYOSHI. *The coöperative movement in Japan.* London: King, 1923. xv, 362 pp. (London School of Economics. Studies No. 69.)

PERRY, CLARENCE A. *The attitude of high school students toward motion pictures.* New York: National Board of Review of Motion Pictures, 1923. 54 pp.

PIGOU, A. C. *Essays in applied economics.* London: King, 1923. vii, 198 pp.

RAWLINSON, ALBERT. *The true principle of money and its application in stabilizing the currencies.* London: King, 1923. 16 pp.

ROBINSON, L. R. *Foreign credit facilities in the United Kingdom. A sketch of post-war development and present status.* New York: Longmans, 1923. 229 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 244.

THE ROCKEFELLER FOUNDATION. *Annual report, 1922.* New York, 1923. xiii, 451 pp.

STAPLES, T. S. *Reconstruction in Arkansas, 1862–1874.* New York: Longmans, 1923. 450 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 245.)

TAFT, D. R. *Two Portuguese communities in New England.* New York: Longmans, 1923. 357 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 241.)

WASHBURNE, G. A. *Imperial control of the administration of justice in the 13 American colonies, 1684–1776.* New York: Longmans, 1923. 191 pp. (Columbia University Studies in History, Economics and Public Law. Whole No. 238.)

MISCELLANEOUS REPRINTS AND PAMPHLETS

MÉTHODED PERMETTANT DE REALISER UNE ECONOMIE DE TRAVAIL DANS LA STATISTIQUE, by Adolph Jensen. Presente a I. S. I., 1923.

SOCIAL AND LABOUR CONDITIONS IN SWEDEN. Reprinted from the Swedish Year Book, 1923.

INFANT MORTALITY AND PREVENTIVE WORK IN NEW ZEALAND, by Robert M. Woodbury. Reprinted from Transactions of the Eleventh Annual Meeting of the American Child Hygiene Association, 1920.

COMPARATIVE STATISTICS OF STATE HOSPITALS FOR MENTAL DISEASES. 1920, by Horatio M. Pollock and Edith M. Furbush. Bureau of Statistics, National Committee for Mental Hygiene.

CANCER AND CIVILIZATION, Address read by Frederick L. Hoffman before the Belgian National Cancer Congress, 1923.

PROGRESS IN CHILD HEALTH, Report of Courtenay Dinwiddie to the directors of the American Child Health Association, 1923.

THE CAUSES FOR THE RECENT DECLINE IN TUBERCULOSIS AND THE OUTLOOK FOR THE FUTURE, Address delivered by Louis I. Dublin, before the Nineteenth Annual Meeting of the National Tuberculosis Association, 1923.

BROADENING THE LIFE SPAN OF THE AMERICAN NEGRO, Metropolitan Life Insurance Company, Statistical Bulletin, Vol. IV. No. 9.

ONE YEAR OF COMMON COLDS AND ASSOCIATED INFECTIONS, Metropolitan Life Insurance Company, Statistical Bulletin, Vol. IV, No. 11.

NOTE ON THE RELIABILITY OF A TEST, WITH SPECIAL REFERENCE TO THE EXAMINATIONS SET BY THE COLLEGE ENTRANCE BOARD, by W. L. Crum. Reprinted from The American Mathematical Monthly.

UNIVERSITY OF ILLINOIS BULLETINS, BUREAU OF EDUCATIONAL RESEARCH.

No. 15. The Constant and Variable Errors of Educational Measurements, by Walter S. Monroe. 1923.

No. 16. "An Annotated Bibliography Dealing with the Classification and Instruction of Pupils to Provide for Individual Differences, by Charles W. Odell. 1923.

No. 17. The Present Status of Written Examinations and Suggestions for their Improvement. 1923.

No. 23. Educational Guidance in High Schools, by Walter S. Monroe. 1923.

THE REGISTRAR-GENERAL'S STATISTICAL REVIEW OF ENGLAND AND WALES FOR THE YEAR 1922. (New Annual Series No. 2.) Tables, Part I, Medical. 1923. Part II, Civil. 1924.